# Final Exam

Monday 12/15 from 3:30-5:30 PM
Elliott Hall (Exact Seating TBD)

Format: 2 hrs, 24 questions

| 12 Questions | Other 12 Questions |
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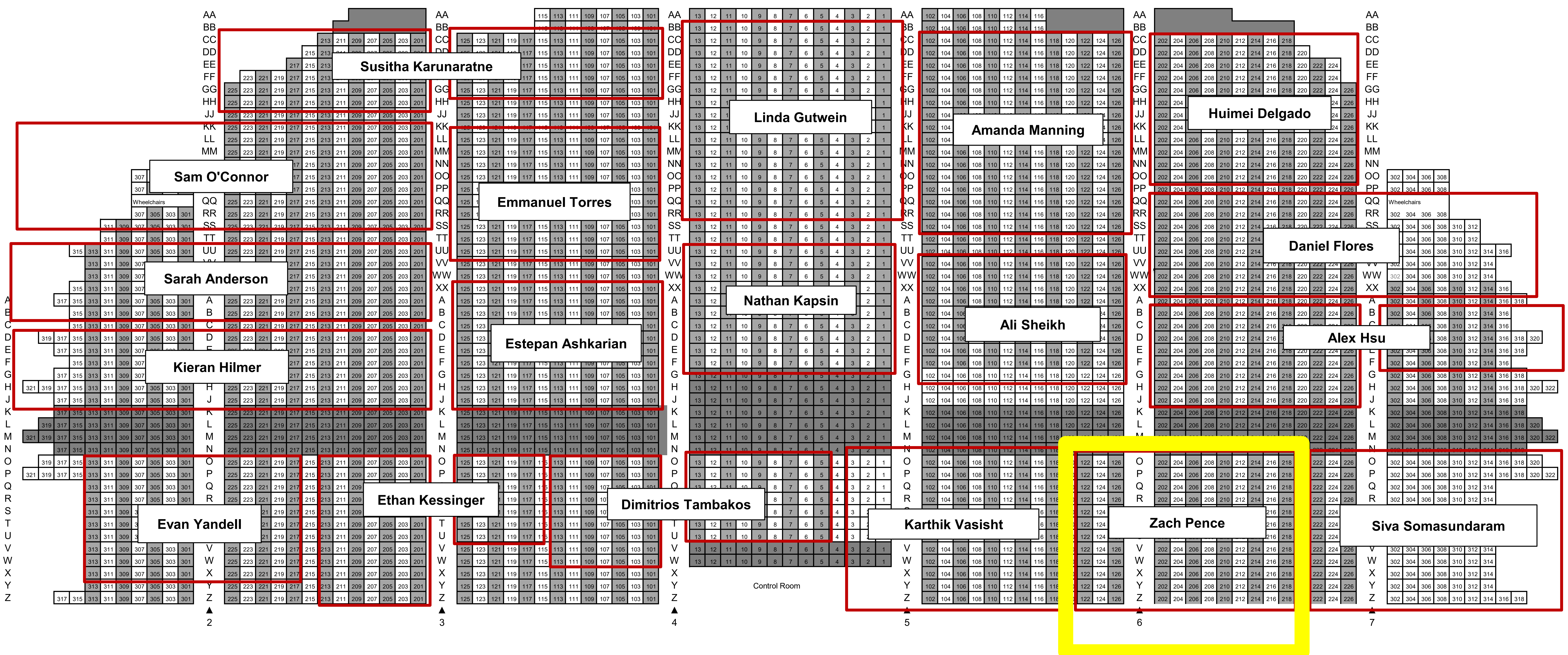
| From Exams 1-3<br>More specifically<br>• 1 question comes <u>directly</u> from Exams 1, 2, and 3 [3 questions total]<br>• 3 questions comes <u>directly</u> from the Exam Problem Sets on Achieve [9 Total] | Covers the content after Exam 3 (Lectures 29-35) |

**MA 16010**
**Final Exam**

**Elliott Hall of Music**
**Purdue University**
**Main Floor**
No seating Rows K, L, M, N - All Sections
No seating Rows G, H, J, K, L, M, N - Center Sections
3561/1700 Stations

**Mon, Dec. 15, 2025**
**3:30 - 5:30 p.m.**

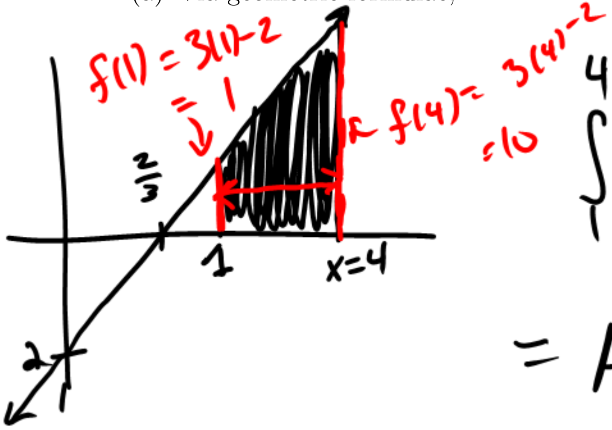


**Problems for Day 3 (Lectures 29-35):** Content after Exam 3; FTC/NCT, Num. Int., Exp. Growth/Decay

1. Compute

$$\int_1^4 (3x-2)\ dx$$

   (a) Via geometric formulae;




$$\int_1^4 (3x-2)\,dx$$

$$= \text{Area of the trapezoid}$$

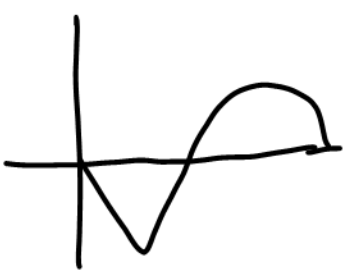
$$= \frac{1}{2}(b_1+b_2)h$$

$$= \frac{1}{2}(1+10)3 = \frac{33}{2}$$

   (b) Verify your answer using the FTC.

$$\int_1^4 (3x-2)\,dx = \left[3\cdot\frac{x^{1+1}}{1+1} - 2x\right]_1^4$$

$$= \left[\frac{3}{2}x^2 - 2x\right]_1^4$$



$$= \left[3\cdot\frac{16}{2} - 2(4)\right] - \left[\frac{3}{2}(1) - 2(1)\right]$$

$$= [24-8] - \left[\frac{3}{2} - 2\right]$$

$$= 16 - \frac{3}{2} + 2 = 16 + \frac{1}{2} = \frac{33}{2}$$

2. If $\int_0^6 f(x)\ dx = 10$ and $\boxed{\int_0^4 f(x)\ dx = 7}$, find $\int_4^6 f(x)\ dx$.

$$\int_0^6 f(x)\,dx = \int_0^4 f(x)\,dx + \int_4^6 f(x)\,dx$$

$$10 = 7 + \int_4^6 f(x)\,dx$$

$$\int_4^6 f(x)\,dx = 10 - 7 = 3$$

3. Compute:

$$\int_2^4 \frac{1+x-x^2}{x^2}\,dx$$

$$\int_2^4 \frac{1+x-x^2}{x^2}\,dx = \int_2^4 \left(\frac{1}{x^2} + \frac{x}{x^2} - \frac{x^2}{x^2}\right) dx$$

$$= \int_2^4 \left(\frac{1}{x^2} + \frac{1}{x} - 1\right) dx = \int_2^4 \left(x^{-2} + \frac{1}{x} - 1\right) dx$$

$$= \left[\frac{x^{-2+1}}{-2+1} + \ln|x| - x\right]_2^4 = \left[-\frac{1}{x} + \ln|x| - x\right]_2^4$$

$$= \left[-\frac{1}{4} + \ln(4) - 4\right] - \left[-\frac{1}{2} + \ln(2) - 2\right]$$

$$= \underbrace{\left[-\frac{1}{4} - 4 + \frac{1}{2} + 2\right]} + \left[\ln(4) - \ln(2)\right]$$

$$= -\frac{7}{4} + \ln(2) \approx -1.0569$$

4. The growth rate of a population is given by:

$$P'(t) = -25\left(200 - e^t\right)$$

where $P(t)$ is the population after $t$ years. How did the population change in its first 3 years?

Ie, we want to know

$$P(3) - P(0) \underset{\text{FTC}}{\overset{\text{Net Change Theorem}}{=\!=}} \int_0^3 P'(t)\,dt \qquad \frac{200x^{0+1}}{0+1}$$

$$= \int_0^3 -25(200 - e^t)\,dt = -25\int_0^3 (200 - e^t)\,dt$$

$$= -25\left[200t - e^t\right]_0^3$$

$$= -25\left[(600 - e^3) - (0 - 1)\right] = -25\left[600 + 1 - e^3\right]$$

$$\approx -14{,}522.86 = P(3) - P(0)$$

Ie, the population decreased by $\approx 14{,}523$ people

5. The **Standard Normal Distribution** is a probability density function given by the function $N_{0,1}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$. Using $n = 4$ trapezoids, approximate the value of:

$$\int_{-1}^{1} N_{0,1}(x)\, dx = \boxed{\frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{-\frac{x^2}{2}}\, dx}$$

Approxamating $\int_{-1}^{1} e^{-\frac{x^2}{2}} dx$


$\Delta x = \frac{b-a}{N} = \frac{1-(-1)}{4} = \frac{2}{4} = \frac{1}{2}$ $\quad f(x) = e^{-\frac{x^2}{2}}$

$T_4 = \frac{1/2}{2}\left[f(-1) + 2f(-\tfrac{1}{2}) + 2f(0) + 2f(\tfrac{1}{2}) + f(1)\right]$

$= \frac{1}{4}\left[e^{-\frac{1}{2}} + 2e^{-\frac{1}{8}} + 2e^{0} + 2e^{-\frac{1}{8}} + e^{-\frac{1}{2}}\right]$

$= \frac{1}{4}\left[0.6065 + 1.7650 + 2 + 1.7650 + 0.6065\right]$

$= \frac{1}{4}\left[6.7430\right] \approx 1.6858 \approx \int_{-1}^{1} e^{-\frac{x^2}{2}} dx$

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$\int_{-1}^{1} N_{0,1}(x)\, dx = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{-\frac{x^2}{2}} dx \approx \frac{1}{\sqrt{2\pi}} T_4$

$\approx \frac{1}{\sqrt{2\pi}}(1.6858) \approx 0.6725$

For context, $\int_{-1}^{1} N_{0,1}(x)\, dx \approx 0.68269$

6. Suppose you deposit \$500 in a savings account, and after 1 year, there is \$531.87 in the account. Assume the interest rate remains constant and no additional deposits or withdrawals are made. How long will it take the balance to reach \$2500?

$$P(t) = P_0 e^{kt}$$

$$P(t) = 500 e^{kt}$$

Determine $k$:

$$531.87 = P(1) = 500 e^{k(1)}$$

$$531.87 = 500 e^{k}$$

$$e^{k} = \frac{531.87}{500}$$

$$k = \ln\left(\frac{531.87}{500}\right) \approx 0.0618$$

$$P(t) = 500 e^{0.0618t}$$

$$2500 = 500 e^{0.0618t}$$

$$5 = e^{0.0618t}$$

$$P(t) = P_0 e^{kt}$$

$$\ln(5) = 0.0618t$$

$$t = \frac{\ln(5)}{0.0618} \approx 26.04 \text{ years}$$

7. Researchers determine that a fossilized bone has 30% of the Carbon-14 ($^{14}_{6}C$) of a live bone. Estimate the age of the bone. Assume a half-life for $^{14}_{6}C$ of 5715 years.

$$P(t) = P_0 e^{kt}$$

Find k:

$$\frac{1}{2} \cdot P_0 = 1 \cdot P_0 e^{5715k}$$

$$\frac{1}{2} = e^{5715k}$$

$$\ln\left(\tfrac{1}{2}\right) = 5715k$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{5715} = -\frac{\ln(2)}{5715}$$

Estimate Age:

$$0.3 \cdot P_0 = 1 P_0 e^{-\frac{\ln(2)}{5715} t}$$

$$0.3 = e^{-\frac{\ln(2)}{5715} t}$$

$$\ln(0.3) = -\frac{\ln(2)}{5715} t$$

$$t = \frac{\ln(0.3)}{-\frac{\ln(2)}{5715}} = \frac{-[\ln(3) - \ln(10)] \cdot 5715}{\ln(2)}$$

$$\approx 9926.76 \text{ years}$$