Final Exam

Monday 12/15 from 3:30-5:30 PM
Elliott Hall (Exact Seating TBD)

Format: 2 hrs, 24 questions

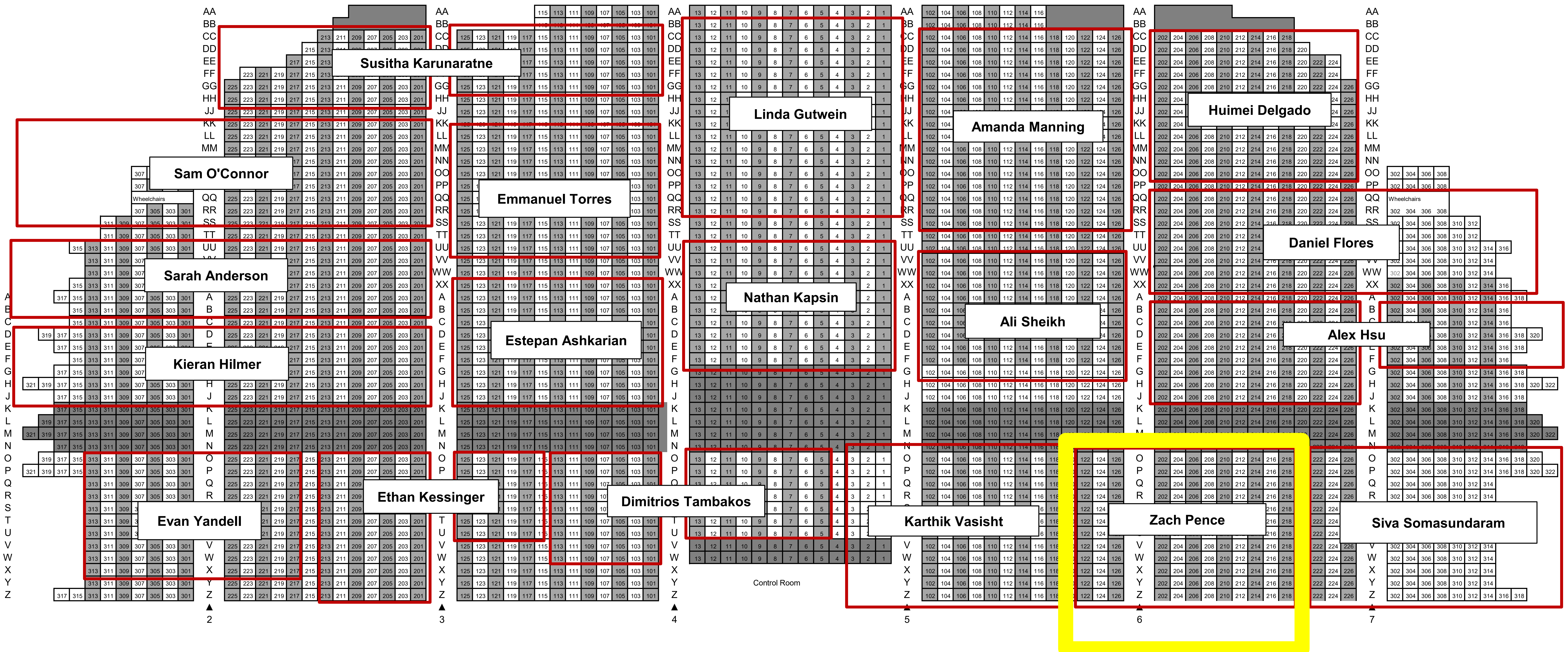
12 Questions	Other 12 Questions
From Exams 1-3 More specifically • 1 question comes <u>directly</u> from Exams 1, 2, and 3 [3 questions total] • 3 questions comes <u>directly</u> from the Exam Problem Sets on Achieve [9 Total]	Covers the content after Exam 3 (Lectures 29-35)

Elliott Hall of Music
Purdue University
Main Floor

No seating Rows K, L, M, N - All Sections
No seating Rows G, H, J, K, L, M, N - Center Sections
3561/1700 Stations

MA 16010
Final Exam

Mon, Dec. 15, 2025
3:30 - 5:30 p.m.

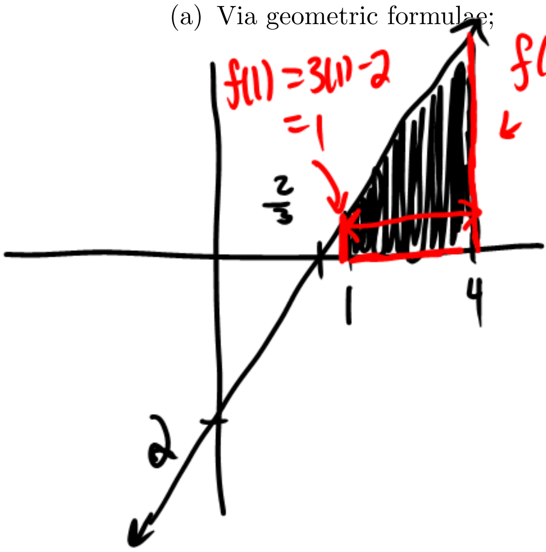


Problems for Day 3 (Lectures 29-35): Content after Exam 3; FTC/NCT, Num. Int., Exp. Growth/Decay

1. Compute

$$\int_1^4 (3x-2)\ dx$$

(a) Via geometric formulae;



$$\int_1^4 (3x-2)dx$$

= Area of the trapezoid

$$= \frac{1}{2}(b_1+b_2)h$$

$$= \frac{1}{2}(1+10)3$$

$$= \frac{33}{2}$$

(b) Verify your answer using the FTC.

$$\int_1^4 (3x-2)\,dx = \left[3\cdot\frac{x^{1+1}}{1+1} - 2\frac{x^{0+1}}{1+1}\right]_1^4$$

$$= \left[\frac{3}{2}x^2 - 2x\right]_1^4 = \left[\frac{3}{2}(16) - 2(4)\right] - \left[\frac{3}{2}(1) - 2(1)\right]$$

$$= [24-8] - [3/2 - 2] = 16 + \frac{1}{2} = \frac{33}{2}$$

2. If $\int_0^6 f(x)\ dx = 10$ and $\int_0^4 f(x)\ dx = 7$, find $\int_4^6 f(x)\ dx$.

$$\underbrace{\int_0^6 f(x)\,dx}_{} = \underbrace{\int_0^4 f(x)\,dx}_{} + \int_4^6 f(x)\,dx$$

$$10 = 7 + \int_4^6 f(x)\,dx$$

$$\int_4^6 f(x)\,dx = 10 - 7 = \boxed{3}$$

3. Compute:

$$\int_2^4 \frac{1+x-x^2}{x^2}\,dx$$

$$\int_2^4 \frac{1+x-x^2}{x^2}\,dx = \int_2^4 \left(\frac{1}{x^2}+\frac{x}{x^2}-\frac{x^2}{x^2}\right)dx$$

$$= \int_2^4 \left(x^{-2}+\frac{1}{x}-1\right)dx = \left[\frac{x^{-2+1}}{-2+1}+\ln|x|-x\right]_2^4$$

$\underbrace{\frac{x^{1+1}}{-1+1}}$

Can't Use Rev. Power Rule when exponent is -1

$$= \left[-\frac{1}{x}+\ln|x|-x\right]_2^4$$

$$= \left[-\frac{1}{4}+\ln(4)-4\right]-\left[-\frac{1}{2}+\ln(2)-2\right]$$

$$= \left[-\frac{1}{4}-4+\frac{1}{2}+2\right]+\left[\ln(4)-\ln(2)\right]$$

$$= -\frac{7}{4}+\ln(2)\approx -1.0569$$

4. The growth rate of a population is given by:

$$P'(t) = -25\left(200 - e^t\right)$$

where $P(t)$ is the population after t years. How did the population change in its first 3 years?

I.e., we want to find

$$P(3) - P(0) \underset{\text{FTC/NCT}}{=} \int_0^3 P'(t)\,dt$$

$$= \int_0^3 -25(200 - e^t)\,dt = -25\int_0^3 (200 - e^t)\,dt$$

$$= -25\left[200t - e^t\right]_0^3$$

$$= -25\left[(600 - e^3) - (0 - 1)\right] = -25\left[601 - e^3\right]$$

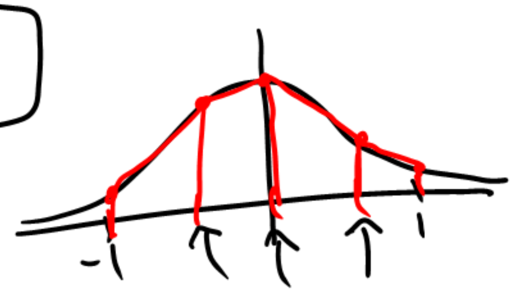
$$\approx -14{,}522.86 = \underbrace{P(3) - P(0)}$$

$$\Rightarrow P(3) < P(0) \Rightarrow \text{Population Shrank}$$

The population decreased by $\approx 14{,}523$ people.

5. The **Standard Normal Distribution** is a probability density function given by the function $N_{0,1}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$. Using $n = 4$ trapezoids, approximate the value of:

$$\int_{-1}^{1} N_{0,1}(x)\ dx = \boxed{\frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{-\frac{x^2}{2}}\ dx}$$



Approximating $\int_{-1}^{1} e^{-\frac{x^2}{2}} dx$

$\Delta x = \frac{b-a}{n} = \frac{1-(-1)}{4} = \frac{1}{2}$

$f(x) = e^{-\frac{x^2}{2}}$

$T_4 = \frac{1/2}{2}\left[f(-1) + 2f(-\tfrac{1}{2}) + 2f(0) + 2f(\tfrac{1}{2}) + f(1)\right]$

$= \frac{1}{4}\left[e^{-\frac{1}{2}} + 2e^{-\frac{1}{8}} + 2e^{0} + 2e^{-\frac{1}{8}} + e^{-\frac{1}{2}}\right]$

$= \frac{1}{4}\left[0.6065 + 1.7650 + 2 + 1.7650 + 0.6065\right]$

$= \frac{1}{4}\left[6.7430\right] \approx 1.6858 \approx \int_{-1}^{1} e^{-\frac{x^2}{2}} dx$

So, $\int_{-1}^{1} N_{0,1}(x)\, dx = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{-\frac{x^2}{2}} dx \approx \frac{1}{\sqrt{2\pi}} \cdot T_4$

$= \frac{1}{\sqrt{2\pi}} \cdot (1.6858) \approx 0.6725$

For context, $\frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{-\frac{x^2}{2}} dx \approx 0.68269$

6. Suppose you deposit \$500 in a savings account, and after 1 year, there is \$531.87 in the account. Assume the interest rate remains constant and no additional deposits or withdrawals are made. How long will it take the balance to reach \$2500?

$$P(t) = P_0 e^{kt}$$

$$P(t) = 500 e^{kt}$$

Find k:

$$531.87 = P(1) = 500 e^{k(1)}$$

$$531.87 = 500 e^{k}$$

$$\frac{531.87}{500} = e^{k}$$

$$0.0618 \approx \ln\left(\frac{531.87}{500}\right) = k$$

$$P(t) = 500 e^{0.0618t}$$

$$2500 = 500 e^{0.0618t}$$

$$5 = e^{0.0618t}$$

$$\ln 5 = 0.0618t$$

$$t = \frac{\ln(5)}{0.0618} \approx 26.04 \text{ years}$$

7. Researchers determine that a fossilized bone has 30% of the Carbon-14 ($^{14}_{6}C$) of a live bone. Estimate the age of the bone. Assume a half-life for $^{14}_{6}C$ of 5715 years.

$$P(t) = P_0 e^{kt}$$

Find k:

$$\frac{1}{2} \cdot P_0 = 1 \cdot P_0 e^{k(5715)}$$

$$\frac{1}{2} = e^{k(5715)}$$

$$\ln\left(\frac{1}{2}\right) = k(5715)$$

$$k = \frac{\ln(1/2)}{5715} = -\frac{\ln(2)}{5715}$$

$$\frac{3}{10} \cdot P_0 = 1 \cdot P_0 e^{-\frac{\ln(2)}{5715}t}$$

$$\frac{3}{10} = e^{-\frac{\ln(2)}{5715}t}$$

$$\ln\left(\frac{3}{10}\right) = -\frac{\ln(2)}{5715}t$$

$$t = \frac{\ln(3/10)}{-\frac{\ln(2)}{5715}} = -\frac{[\ln(3) - \ln(10)] \cdot 5715}{\ln(2)}$$

$$= \frac{[\ln(10) - \ln(3)] \cdot 5715}{\ln(2)} \approx 9926.76 \text{ years}$$