Goal: Compute derivatives of derivatives. Interpret the second and third derivative in a physics context.
Summary:

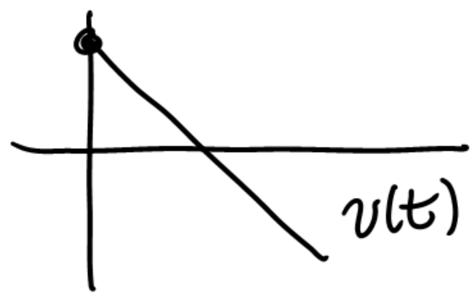
$$\frac{d^n f}{dx^n} = \frac{d}{dx}\left(\frac{d^{n-1}f}{dx^{n-1}}\right) \qquad f^{(n)}(x) = [f^{(n-1)}(x)]'$$
$$a(t) = v'(t) = s''(t) \qquad j(t) = a'(t) = v''(t) = s'''(t)$$

Recall that a ball being tossed straight into the air can be modeled using a parabola, say

$$s(t) = -4.9t^2 + 7t + 6$$

We called $\frac{ds}{dt} = v(t)$ the velocity

$$v(t) = -9.8t + 7$$



Let's take $\frac{dv}{dt}$, $\frac{dv}{dt} = -9.8 =$ The acceleration of the ball

Def For a fcn f,

$$\frac{d}{dx}\left(\frac{df}{dx}\right) = \frac{d^2f}{dx^2}$$

$$[f'(x)]' = f''(x)$$

is called the second derivative of f w.r.t. x

<u>Def</u> For an object with position fcn $s(t)$,

$$a(t) \overset{\text{def}}{=} \frac{dv}{dt} = \frac{d^2s}{dx^2}$$

is called the <u>acceleration</u> fcn.

Ex1/ The position of a Particle is given by

$$s(t) = t^3 - 6t^2 + 9t$$

ⓐ Find $v(t)$ and $a(t)$

$$v(t) = s'(t) = 3t^2 - 12t + 9$$

$$a(t) = v'(t) = 6t - 12$$

ⓑ If t is measured in seconds, $s(t)$ is in meters, then find $v(4)$ and $a(4)$

$$v(4) = 3(4)^2 - 12(4) + 9 = 9 \frac{m}{s}$$

$$a(4) = 6(4) - 12 = 12 \frac{m/s}{s} = 12 \frac{m}{s^2}$$

Ex2/ Same setup as Ex1, but

$$s(t) = \cos 2t$$

ⓐ Find $v(t)$, $a(t)$

$$v(t) = s'(t) = (2) -\sin 2t = -2 \underbrace{\sin 2t}$$

$$a(t) = v'(t) = -2 \cos 2t\,(2) = -4\cos 2t$$

ⓑ Find $a\left(\frac{\pi}{2}\right)$ $\oplus$

$$a\left(\frac{\pi}{2}\right) = -4\cos 2\left(\frac{\pi}{2}\right) = -4\cos\pi = (-4)(-1)$$
$$= 4\ \frac{m}{s^2}$$

Ex 3/ Given $s(t) = \dfrac{t}{1+t^2}$ $(t \geq 0)$, when is the ~~velocity constant?~~ Acceleration 0?

~~To determine when velocity is constant, find when the acceleration is 0.~~

$$v(t) = \frac{(1)(1+t^2) - t(2t)}{(1+t^2)^2} = \frac{1+t^2-2t^2}{(1+t^2)^2}$$
$$= \frac{1-t^2}{(1+t^2)^2}$$

$$a(t) = v'(t) = \frac{(-2t)(1+t^2)^2 - (1-t^2)(2)(1+t^2)\cancel{(2t)}}{(1+t^2)^4}$$

$$= \frac{-2t(1+t^2)\left[(1+t^2) + 2(1-t^2)\right]}{(1+t^2)^4}$$

$$= \frac{-2t(1+t^2)\left[3-t^2\right]}{\underbrace{(1+t^2)^4}_{\text{Never equals } 0}} \overset{\text{set}}{=} 0$$

$$\Rightarrow -2t\,(1+t^2)\,[3-t^2] = 0$$

Either,

$-2t = 0$ OR $1+t^2=0$ OR $3-t^2=0$

$\boxed{t=0}$ Impossible $t^2 = 3$

$t \geq 0 \longrightarrow t = \pm\sqrt{3}$

$\boxed{t = \sqrt{3}}$

$[v(t)]' = c'$

$a(t) = 0$

Ex 4/ If $f(x) = 2\cos x + 3\sin x$, what is $f(0)$, $f'(0)$, $f''(0)$?

$f(x) = 2\cos x + 3\sin x \longrightarrow f(0) = 2$

$f'(x) = -2\sin x + 3\cos x \longrightarrow f'(0) = 3\cos 0$
$= 3$

$f''(x) = \underbrace{-2\cos x}-3\sin x \longrightarrow f''(0) = -2\cos 0$
$= -2$

Higher Order Derivatives

NOTATION:

3rd Derivative: $\frac{d}{dx}\left(\frac{d^2f}{dx^2}\right) \overset{\text{def}}{=} \frac{d^3f}{dx^3}$

$[f''(x)]' = f'''(x)$

4^{th} Derivative: $$\frac{d^4f}{dx^4} \stackrel{def}{=} \frac{d}{dx}\left(\frac{d^3f}{dx^3}\right)$$

$$[f'''(x)]' = f''''(x)$$

n-th derivative: $$\frac{d^nf}{dx^n} \stackrel{def}{=} \frac{d}{dx}\left(\frac{d^{n-1}f}{dx^{n-1}}\right)$$

$$f^{(n)}(x) = [f^{(n-1)}(x)]'$$

<u>Def</u> For a position fcn $s(t)$,

$$s'''(t) = v''(t) = a'(t) \stackrel{def}{=} j(t)$$

$j(t)$ is the jerk function.

Ex 5/ If $f^{(9)}(x) = 6\sec(2x-5)$, what's $f^{(10)}(x)$?

$$f^{(10)}(x) = [f^{(9)}(x)]' = [6\sec(2x-5)]'$$

$$= 6\sec(2x-5)\tan(2x-5)(2)$$

$$= 12\sec(2x-5)\tan(2x-5)$$

Ex 6/ Examine the various derivatives for $\sin x$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$$\sin x \to \cos x \to -\sin x \to -\cos x \to \sin x$$

Ex 7/ Repeat for $x^4 + x^3 + x^2 + x + 1$

$f(x) = x^4 + x^3 + x^2 + x + 1$

$\downarrow$

$f'(x) = 4x^3 + 3x^2 + 2x + 1$

$\downarrow$

$f''(x) = 12x^2 + 6x + 2$

$\downarrow$

$f'''(x) = 24x + 6 \longrightarrow f^{(4)}(x) = 24$

$\uparrow$

$f^{(5)}(x) = 0$

Ex 8/ Repeat for e^{2x}

$f(x) = e^{2x}$

$\downarrow$

$f'(x) = 2e^{2x}$

$\downarrow$

$f''(x) = 2 \cdot 2e^{2x} = 4e^{2x} \Rightarrow \cdots \; f^{(n)}(x) = 2^n e^{2x}$