Goal: Compute derivatives of derivatives. Interpret the second and third derivative in a physics context.
Summary:

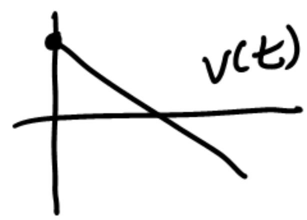
$\frac{d^n f}{dx^n} = \frac{d}{dx}\left(\frac{d^{n-1} f}{dx^{n-1}}\right)$	$f^{(n)}(x) = [f^{(n-1)}(x)]'$
$a(t) = v'(t) = s''(t)$	$j(t) = a'(t) = v''(t) = s'''(t)$

Recall that a ball tossed straight into the air can be modeled by a parabola, say

$$s(t) = -4.9t^2 + 7t + 6$$

We called $\frac{ds}{dt} = v(t)$ the velocity

$$v(t) = -9.8t + 7$$



So, $\frac{dv}{dt} = -9.8 \overset{\text{def}}{=}$ The acceleration of the ball

Def The quantity

$$\frac{d}{dx}\left(\frac{df}{dx}\right) = \frac{d^2 f}{dx^2}$$

$$[f'(x)]' = f''(x)$$

is called the second derivative of f w.r.t. x

Def The acceleration of an object with position function $s(t)$ is the fcn $a(t)$

$$a(t) \overset{\text{def}}{=} \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

Ex1/ The position of particle is given by

$$s(t) = t^3 - 6t^2 + 9t$$

(a) Find $v(t)$ and $a(t)$

$$v(t) = s'(t) = 3t^2 - 12t + 9$$

$$a(t) = v'(t) = 6t - 12$$

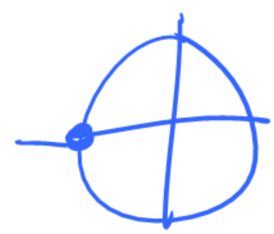
(b) If t is measured in seconds and $s(t)$ is measured in meters, find $v(4)$ and $a(4)$

$$v(4) = 3(4)^2 - 12(4) + 9 = 9\,\frac{m}{s}$$

$$a(4) = 6(4) - 12 = 12\,\frac{m/s}{s} = 12\,\frac{m}{s^2}$$

Ex2/ Same setup as Ex1, but

$$s(t) = \cos 2t$$



(a) Find $v(t)$ and $a(t)$

$$v(t) = s'(t) = (-\sin 2t)(2) = -2\sin 2t$$

$$a(t) = v'(t) = -2\cos 2t\,(2) = -4\cos 2t$$

(b) Find $a\left(\frac{\pi}{2}\right)$

$$a\left(\frac{\pi}{2}\right) = -4\cos 2\left(\frac{\pi}{2}\right) = -4\cos\pi = (-4)(-1)$$

$$= 4\,\frac{m}{s^2}$$

Ex 3/ Given $s(t)=\frac{t}{1+t^2}$ $(t \geq 0)$, when is the acceleration 0?

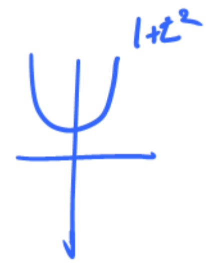
$$v(t)=s'(t)=\frac{(1)(1+t^2)-(t)(2t)}{(1+t^2)^2}=\frac{1+t^2-2t^2}{(1+t^2)^2}$$

$$=\frac{1-t^2}{(1+t^2)^2}$$

$$a(t)=v'(t)=\frac{(-2t)(1+t^2)^2-(1-t^2)(2)(1+t^2)(2t)}{(1+t^2)^4}$$

$$=\frac{-2t(1+t^2)\left[1+t^2+\overbrace{2(1-t^2)}^{2-2t^2}\right]}{(1+t^2)^4}$$

$$=\frac{-2t(1+t^2)[3-t^2]}{\underbrace{(1+t^2)^4}_{\text{Never equals 0}}} \overset{set}{=} 0$$



$$\Rightarrow -2t(1+t^2)[3-t^2]=0$$

$\Rightarrow$ Either $-2t=0$ OR $1+t^2=0$ OR $3-t^2=0$

$\boxed{t=0}$ impossible $t^2=3$

$t \geq 0 \rightarrow t=\pm\sqrt{3}$

$\boxed{t=\sqrt{3}}$

$\boxed{a(t)\overset{set}{=}0, \text{ solve for } t}$

Ex 4/ If $f(x) = 2\sin x + 3\cos x$, What is $f(0)$, $f'(0)$, and $f''(0)$?

$f(x) = 2\sin x + 3\cos x \rightarrow f(0) = 3$

$f'(x) = 2\cos x - 3\sin x \rightarrow f'(0) = 2$

$f''(x) = -2\sin x - 3\cos x \rightarrow f''(0) = -3$

Higher Order Derivatives

NOTATION: 3rd Derivative: $\frac{d}{dx}\left(\frac{d^2 f}{dx^2}\right) = \frac{d^3 f}{dx^3}$

$[f''(x)]' = f'''(x)$

4th Derivative: $\frac{d}{dx}\left(\frac{d^3 f}{dx^3}\right) = \frac{d^4 f}{dx^4}$

$[f'''(x)]' = f''''(x)$

nth Derivative: $\frac{d}{dx}\left(\frac{d^{n-1} f}{dx^{n-1}}\right) = \frac{d^n f}{dx^n}$

$[f^{(n-1)}(x)]' = f^{(n)}(x)$

Ex 5/ If $f^{(9)}(x) = 6\sec(\underbrace{2x-5})$, what is $f^{(10)}(x)$?

$f^{(10)}(x) = [f^{(9)}(x)]' = 6\sec(2x-5)\tan(2x-5)(2)$

$= 12\sec(2x-5)\tan(2x-5)$

Ex 6/ Examine the various derivatives of $\sin x$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$$\begin{tikzcd} & \sin x \arrow[dr] & \\ -\cos x \arrow[ur] & & \cos x \arrow[dl] \\ & -\sin x \arrow[ul] & \end{tikzcd}$$

Ex 7/ Repeat $x^4 + x^3 + x^2 + x + 1$

$$f(x) = x^4 + x^3 + x^2 + x + 1$$
$$\downarrow$$
$$f'(x) = 4x^3 + 3x^2 + 2x + 1$$
$$\downarrow$$
$$f''(x) = 12x^2 + 6x + 2$$
$$\downarrow$$
$$f'''(x) = 24x + 6$$
$$\downarrow$$
$$f^{(4)}(x) = 24$$
$$\downarrow$$
$$f^{(5)}(x) = 0$$

Ex 8/ Repeat for e^{2x}

$$\underset{e^{2x}}{\overset{f(x)}{\shortparallel}} \longrightarrow \underset{2e^{2x}}{\overset{f'(x)}{\shortparallel}} \longrightarrow \underset{2(2e^{2x}) = 4e^{2x}}{\overset{f''(x)}{\shortparallel}} \longrightarrow \underset{8e^{2x}}{\overset{f'''(x)}{\shortparallel}} \rightarrow \cdots\cdots \rightarrow \underset{2^n e^{2x}}{\overset{f^{(n)}(x)}{\shortparallel}} \longrightarrow$$