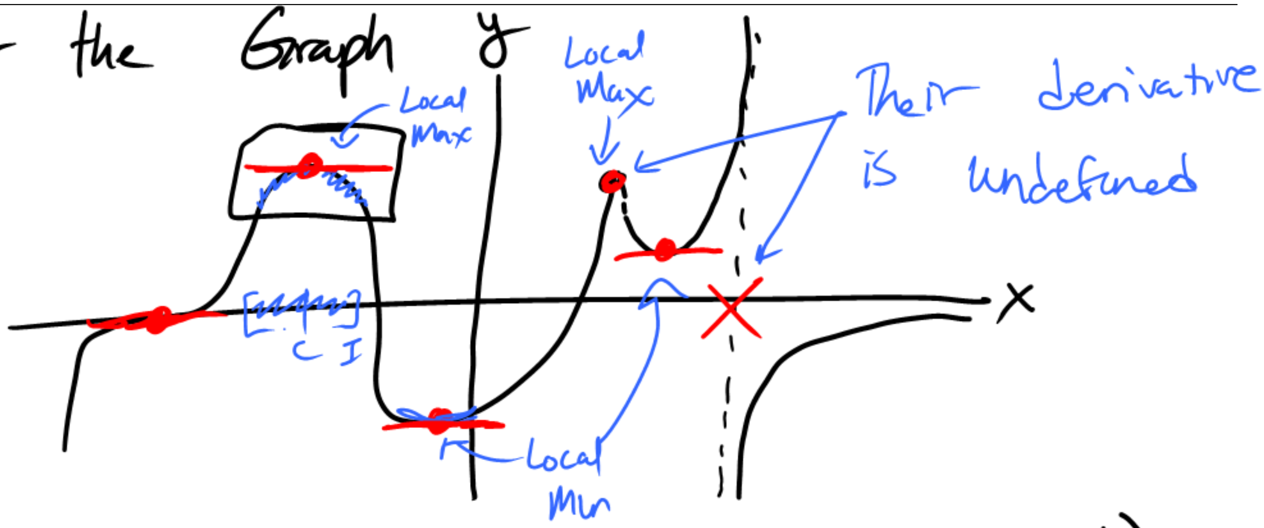
Consider the Graph

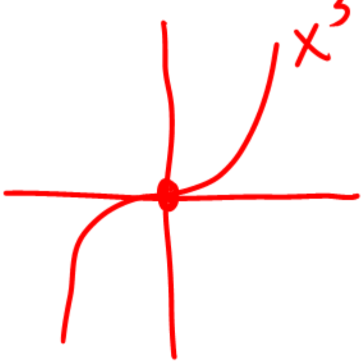


Def A function f has a <u>relative</u> (or <u>local</u>) maximum if there is an interval I containing c such that $f(x) \leq f(c)$ for all x in I.

Def For relative minumums, replace $f(x) \leq f(c)$ with $f(c) \leq f(x)$

Thm If f has a local max/min at $x = c$, then either $f'(c) = 0$ or $f'(c)$ is undefined.

NOTE: The reverse is not true



$f(x) = x^3$

$f'(x) = 3x^2$

$f'(0) = 0$, but 0 is not the location of a local max/min

Def If $f'(c)$ is 0 or undefined, we say c is a <u>critical number</u> of f. (Provided c is in the domain)

Ex1/ Is $x=\pi$ a critical number of $f(x)=8\sin(x-\frac{\pi}{2})$

$f'(x)=8\cos(x-\frac{\pi}{2})$

$f'(\pi)=8\cos(\pi-\frac{\pi}{2})=8\cos\frac{\pi}{2}=0$ ✓ Yes

Ex2/ Is $x=\frac{\pi}{2}$ a critical number of $f(x)=8\sin(x-\frac{\pi}{2})$

$f'(x)=8\cos(x-\frac{\pi}{2})$

$f'(\frac{\pi}{2})=8\cos(\frac{\pi}{2}-\frac{\pi}{2})=8\cos(0)=8$ ✗ No

Ex3/ Find all critical numbers of $f(x)=x^3-75x$

$f'(x)=3x^2-75$

When $f'(x)=0$:

$3x^2-75 \underset{\text{set}}{=} 0$

$3(x^2-25)=0$

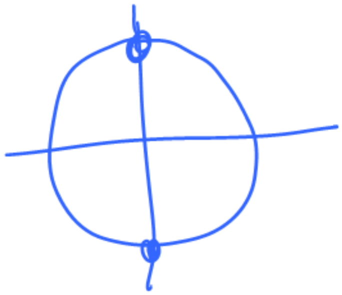
$3(x+5)(x-5)=0 \Longrightarrow \boxed{x=-5,5}$

Ex4/ Find all critical numbers of $f(x)=\tan x$ on the interval $[-\pi,\pi)$

$f(x)=\tan x \longrightarrow f'(x)=\sec^2 x=\frac{1}{\cos^2 x}$

$f'(x)$ is never 0; however, $f'(x)$ is undefined

$\cos^2 x \underset{\text{set}}{=} 0$



$\cos x = 0$

On the interval $[-\pi, \pi)$

$x = \cancel{-\frac{\pi}{2}, \frac{\pi}{2}}$ Not in the domain, not critical nums

Ex 5/ Find all critical numbers of $f(x) = \frac{\ln(x)}{x}$

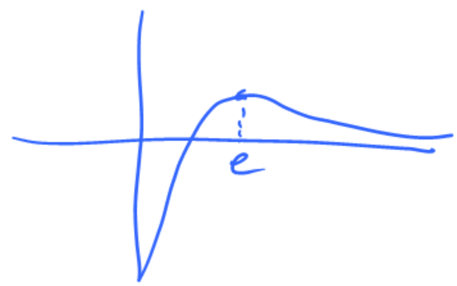
$$f'(x) = \frac{\left(\frac{1}{x}\right)\cdot x - \ln(x)\cdot(1)}{x^2} = \frac{1 - \ln(x)}{x^2}$$

$f'(x)$ is undefined:

$$x^2 = 0 \Longrightarrow \boxed{x = 0}$$ ← Not in the domain, not crit. nums.

$f'(x)$ is 0:

$$1 - \ln(x) = 0$$
$$\ln(x) = 1$$
$$\boxed{x = e}$$



f has critical numbers at $x = 0, e$

Ex 6/ Find the critical numbers for $f(x) = xe^x$

$$f'(x) = [x]'e^x + x[e^x]' = e^x + xe^x = (1+x)e^x$$

$f'(x)$ undefined: Never

$f'(x) = 0$:

$$(1+x)e^x \underset{=}{\text{set}} 0$$

Either: $1+x=0$ OR $e^x = 0$

$\boxed{x=-1}$ Never

Ex 9/ $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12 \underset{=}{\text{set}} 0$

$$6(x^2 - x - 2) = 0$$

$$6(x-2)(x+1) = 0$$

$\Rightarrow$ Either $x-2=0$ OR $x+1=0$

$\boxed{x=2}$ $\boxed{x=-1}$

Quadratic Formula:

$$ax^2+bx+c=0 \Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Ex 10/ $f(x) = x^3 + 5x^2 + 7x$

$$f'(x) = \underbrace{3}_{a}x^2 + \underbrace{10}_{b}x + \underbrace{7}_{c}$$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(3)(7)}}{2(3)} = \frac{-10 \pm \sqrt{100-84}}{6} = \frac{-10 \pm \sqrt{16}}{6}$$

$$= \frac{-10 \pm 4}{6} \Rightarrow \quad x = -\frac{14}{6}, -\frac{6}{6}$$

$$\Rightarrow x = -\frac{7}{3}, -1$$