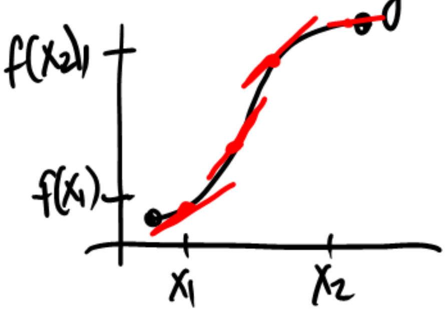
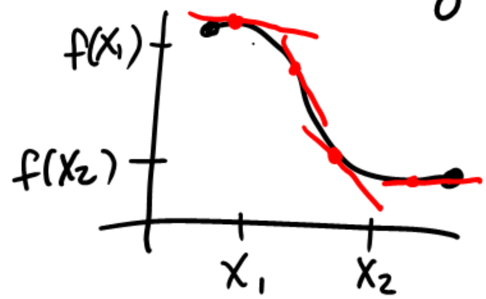
Goal: Use the 1st derivative to determine properties of the original function.

Increasing vs Decreasing



If $x_1 < x_2$, $f(x_1) < f(x_2)$



If $x_1 < x_2$, $f(x_2) < f(x_1)$

<u>Theorem (Inc./Dec. Test)</u> Let f be a differentiable function on an open interval I.

① If $f'(x) > 0$ for every x in I, f is increasing on I.

② If $f'(x) < 0$ for every x in I, f is decreasing on I.

Ex/ When is the function $\frac{x^2}{1+x^2}$ increasing and decreasing?

<u>Step1</u> Determine the critical numbers of f

$$f'(x) = \frac{2x(1+x^2) - x^2(2x)}{(1+x^2)^2} = \frac{2x}{\underbrace{(1+x^2)^2}_{>0}} \overset{\text{set}}{=} 0$$

$2x = 0 \rightarrow \boxed{x = 0}$

<u>Step2</u> Use the critical number(s) to divide the x-axis into regions to determine the sign of f'

$\frac{2x}{(1+x^2)^2}$

Test Point	-100	100
Sign of $2x$	$-$	$+$
Sign of $(1+x^2)^2$	$+$	$+$
Sign of f'	$-$	$+$

f has a local min at $x=0$

Conclusion: f is increasing on $(0,\infty)$, while
f is decreasing on $(-\infty,0)$

Ex 2/ Repeat for $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Step 1 Find crit. nums.

$$f'(x) = 12x^3 - 12x^2 - 24x = 12x(x-2)(x+1) \overset{\text{set}}{=} 0$$

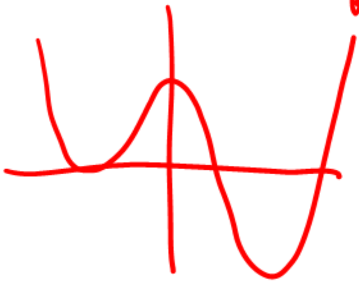
Either, $12x = 0$ $\boxed{x=0}$ OR $x+1=0$ $\boxed{x=-1}$ OR $x-2=0$ $\boxed{x=2}$

Step 2 Make sign chart $12x(x-2)(x+1)$

Test Point	-100	$-\frac{1}{2}$	1	100
Sign of $12x$	$-$	$-$	$+$	$+$
Sign of $x-2$	$-$	$-$	$-$	$+$
Sign of $x+1$	$-$	$+$	$+$	$+$

Sign of f' | $-$ | $+$ | $-$ | $+$

Rel. Min at $x=-1$ Rel. Max at $x=0$ Rel. Min at $x=2$



Conclusion: f is increasing on $(-1,0)\cup(2,\infty)$

f is decreasing on $(-\infty,-1)\cup(0,2)$

Ex3/ Repeat for x^3

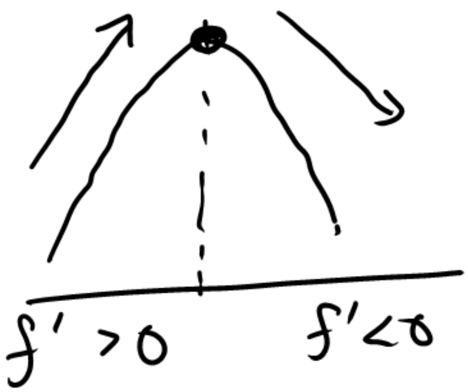
$f'(x)=3x^2 \overset{\text{set}}{=} 0 \implies \boxed{x=0}$

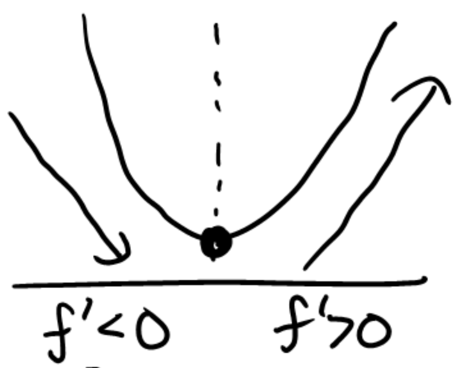
Test Points		-100	0	100
Sign of f'		$+$		$+$

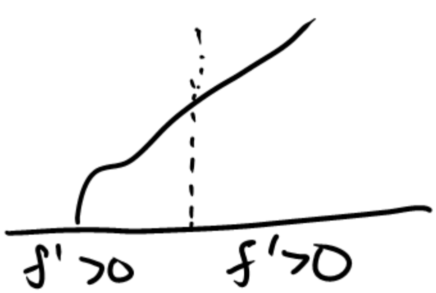
Neither max or min

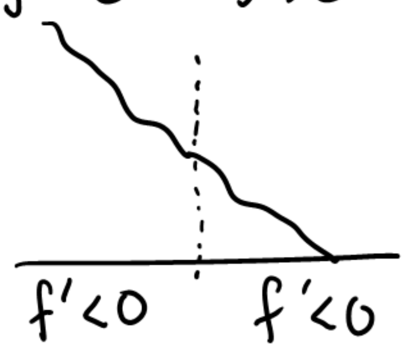
f is always increasing

The I/D Test gives us a way to determine the locations of relative max/mins









Thm (1st Derivative Test) Let c be a critical number of a differentiable function f.

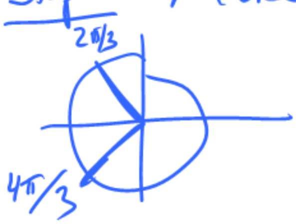
1. If f' switches from positive to negative at c, then f has a relative max at c.
2. If f' switches from negative to positive at c, then f has a rel. min. at c.
3. If f' does not switch signs at c, f has neither a rel. max nor min. at c.

Ex 4/ The critical values of $g(x) = x + 2\sin x$ on $(0, 2\pi)$ are $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Determine the locations of the rel. max/min on the interval $(0, 2\pi)$

Step 1 Determine crit. nums.

$$g'(x) = 1 + 2\cos x$$

Step 2 Make Sign Chart



$0 \qquad \frac{2\pi}{3} \qquad \frac{4\pi}{3} \qquad 2\pi$

Test Points	$\pi/2$	π	$\frac{3\pi}{2}$
Sign of g'	$g'(\frac{\pi}{2}) = 1+2\cdot 0 = 1$ $+$	$g'(\pi) = 1+2(-1) = -1$ $-$	$g'(\frac{3\pi}{2}) = 1+2\cdot 0 = 1$ $+$
Results of I/D Test	Inc	Dec	Inc

Step 3 Determine locations of relative extrema

Conclusion: g has a relative max at $x = \frac{2\pi}{3}$, while g has a rel. min. at $x = \frac{4\pi}{3}$

Ex5/ Repeat for $f(x) = x^4 - 6x^2$

Step 1 Determine crit. nums.

$$f'(x) = 4x^3 - 12x = 4x(x^2-3) \overset{\text{set}}{=} 0$$

Either $4x = 0$ OR $x^2 - 3 = 0$

$\boxed{x = 0}$ $\qquad$ $\boxed{x = \pm\sqrt{3}}$

Step 2 Make sign chart

$-\sqrt{3}$ ≈ -1.7 $\qquad$ 0 $\qquad$ $\sqrt{3}$

Test Points	-100	-1	1	100
Sign of $4x$	$-$	$-$	$+$	$+$
Sign of x^2-3	$+$	$(-1)^2 - 3 = 1-3 = -2$ $-$	$-$	$+$
Sign of f'	$-$	$+$	$-$	$+$
Results of I/D Test	Dec	Inc	Dec	Inc

Min $\qquad$ Max $\qquad$ Min