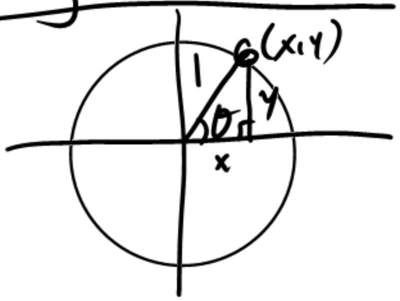
Lecture 1: Review Of Pre Calculus

Trig Functions:



$$\sin\theta = \frac{opp}{hyp} = y$$

$$\cos\theta = \frac{adj}{hyp} = x$$

$$\tan\theta = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

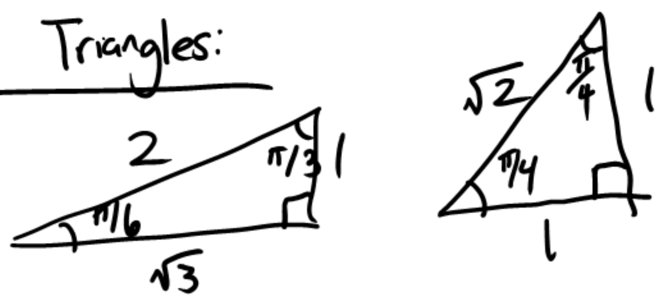
Reciprocal Identities:

$$\frac{1}{\sin\theta} = \csc\theta \qquad \frac{1}{\tan\theta} = \cot\theta = \frac{\cos\theta}{\sin\theta}$$

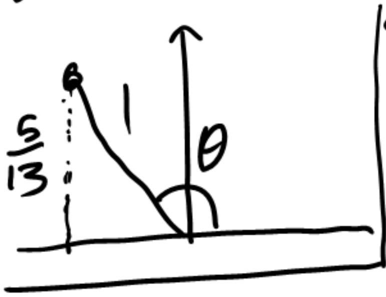
$$\frac{1}{\cos\theta} = \sec\theta$$

Pythagorean Identity: $x^2 + y^2 = 1 \longrightarrow \boxed{\sin^2\theta + \cos^2\theta = 1}$

Special Right Triangles:



Ex/ If $\sin\theta = \frac{5}{13}$ and θ is in the 2^{nd} Quadrant, what are the values of the remaining trig fcns?



Sol/ $\sin\theta = \frac{5}{13}$. Need to find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{5}{13}\right)^2 + \cos^2\theta = 1$$

$$\frac{25}{169} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{25}{169} = \frac{169}{169} - \frac{25}{169} = \frac{144}{169}$$

$$\cos\theta = \pm\sqrt{\frac{144}{169}} = \pm\frac{12}{13}$$

Because $\frac{\pi}{2} < \theta < \pi$, $\boxed{\cos\theta = -\frac{12}{13}}$

$$\sin\theta = 5/13$$

$$\cos\theta = -12/13$$

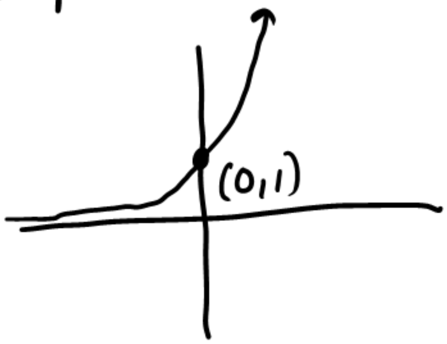
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5/13}{-12/13} = -5/12$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{5}{13}} = \frac{13}{5}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-12/13} = -13/12$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-5/12} = -12/5$$

Exponential Functions: $y = e^x$; $e \approx 2.71828\ldots$



Properties:

- $e^0 = 1$
- $e^x > 0$ for all x
- Strictly increasing
- $e^a e^b = e^{a+b}$
- $e^a / e^b = e^{a-b}$
- $(e^a)^b = e^{ab}$

Ex/ Simplify the following

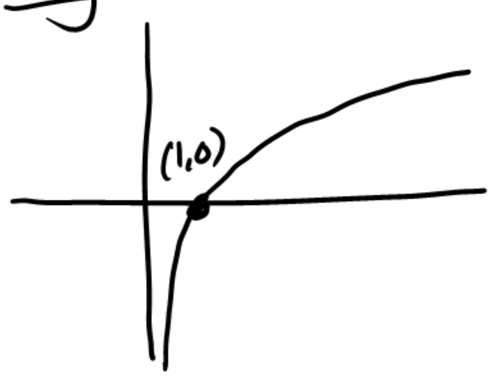
① $e^x e^{-5} = e^{x+(-5)} = e^{x-5}$

② $\dfrac{e^{5x}}{e^{2x}} = e^{5x-2x} = e^{3x}$

③ $(e^x)^2 \neq e^{x^2}$
$= e^{2x}$

④ $\dfrac{e^{2x}e^{3}}{e^x} = \dfrac{e^{2x+3}}{e^x} = e^{(2x+3)-x} = e^{x+3}$

Logarithms $y = \ln(x) = \log_e(x) =$ the exponent where $e^y = x$



Properties

- $\log(1) = 0$
- $\log(ab) = \log(a) + \log(b)$
- $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
- $\log(a^b) = b\log(a)$

e^x and $\ln(x)$ are inverses. That is,

$e^{\ln(x)} = x$

$\ln(e^x) = x$

Ex/ Simplify

① $\ln(e) = 1$

② $\ln(e^{2x}) = 2x\ln(e) = 2x$

③ $e^{18+\ln(20x)} = e^{18}e^{\ln(20x)}$
$= (20x)e^{18}$

Ex/ Solve for x in

$$\ln(x^2) = 5$$

$$e^{\ln(x^2)} = e^5$$

$$x^2 = e^5$$

$$x = \pm\sqrt{e^5} = \pm e^{\frac{5}{2}}$$