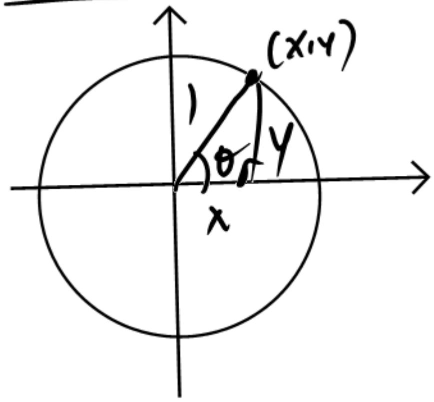
Lecture 1: Review Of Pre Calculus



Trig Functions:

$$\sin\theta = \frac{opp}{hyp} = y$$

$$\cos\theta = \frac{adj}{hyp} = x$$

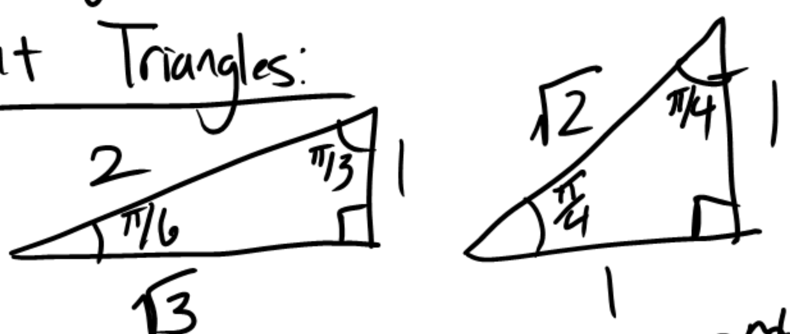
$$\tan\theta = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

Reciprocal Functions:

$$\frac{1}{\sin\theta} = \csc\theta \;;\; \frac{1}{\cos\theta} = \sec\theta \;;\; \frac{1}{\tan\theta} = \cot\theta$$

Pythagorean Identity: $x^2 + y^2 = 1 \longrightarrow \boxed{\sin^2\theta + \cos^2\theta = 1}$

Special Right Triangles:



Ex/ If $\sin\theta = \frac{5}{13}$ and θ is in the 2^{nd} quadrant, then what is the value of the remaining trig fcns?

$\sin\theta = \frac{5}{13} = y$. Need to find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{5}{13}\right)^2 + \cos^2\theta = 1$$

$$\frac{25}{169} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\cos^2\theta = \frac{144}{169}$$

$$\cos\theta = \pm\sqrt{\frac{144}{169}} = \pm\frac{12}{13}$$

Since θ is in the 2^{nd} Quadrant, $\boxed{\cos\theta = \frac{-12}{13}}$

$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{-12}{13}$

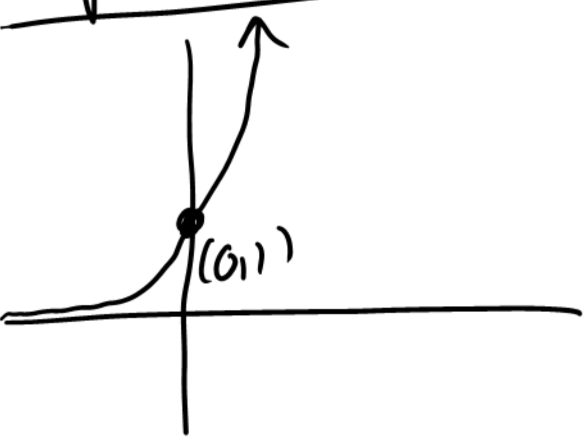
$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5/13}{-12/13} = -5/12$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{5/13} = 13/5$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-12/13} = -13/12$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-5/12} = -12/5$

Exponential Functions $y = e^x$; $e \approx 2.718281\ldots$



Properties:

- $e^0 = 1$
- $e^x > 0$ for all x
- Strictly increasing
- $e^a e^b = e^{a+b}$
- $e^a / e^b = e^{a-b}$
- $(e^a)^b = e^{a \cdot b}$

Ex/ Simplify the Following

① $e^x e^{-5} = e^{x+(-5)} = e^{x-5}$

② $\frac{e^{3x}}{e^{5x}} = e^{3x-5x} = e^{-2x} = \frac{1}{e^{2x}}$

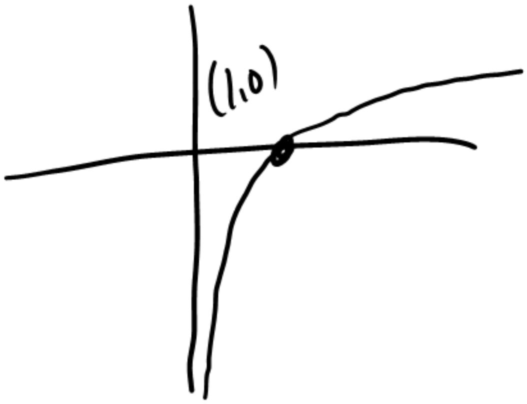
③ $(e^x)^2 \neq e^{x^2}$

$(e^x)^2 = e^{2x}$

④ $\frac{e^{2x} e^3}{e^x} = \frac{e^{2x+3}}{e^x}$

$= e^{(2x+3)-x} = e^{x+3}$

Logarithms $y = \ln(x) = \log_e(x) =$ the exponent such that $e^y = x$



Properties:

- $\ln(1) = 0$
- e^x and $\ln(x)$ are inverses

$$e^{\ln(x)} = x$$

$$\ln(e^x) = x$$

- $\ln(ab) = \ln(a) + \ln(b)$
- $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
- $\ln(a^{b}) = b\ln(a)$

Ex/ Simplify the following:

1. $\ln(e) = 1$
2. $\ln(e^{2x}) = 2x[\ln(e)] = 2x$
3. $e^{18+\ln(20x)} = e^{18} e^{\ln(20x)} = e^{18}(20x)$

$$\boxed{\begin{aligned} e^a e^b &= e^{a+b} \\ (e^a)^b &= e^{ab} \end{aligned}}$$

4. $e^{6\ln(\sqrt{5x})} = e^{6\ln([5x]^{\frac{1}{2}})}$
$= e^{6\left(\frac{1}{2}\right)\ln(5x)} = e^{3\ln(5x)}$
$= \left[e^{\ln(5x)}\right]^3 = (5x)^3 = 125x^3$

5. $\ln\left(\frac{xy}{z}\right) = \ln(xy) - \ln(z) = \ln(x) + \ln(y) - \ln(z)$

Ex/ Solve for x in

$$\ln(x^2) = 5$$

$$e^{\ln(x^2)} = e^5$$

$$x^2 = e^5$$

$$x = \pm\sqrt{e^5}$$

$$= \pm(e^5)^{\frac{1}{2}} = \pm e^{\frac{5}{2}}$$