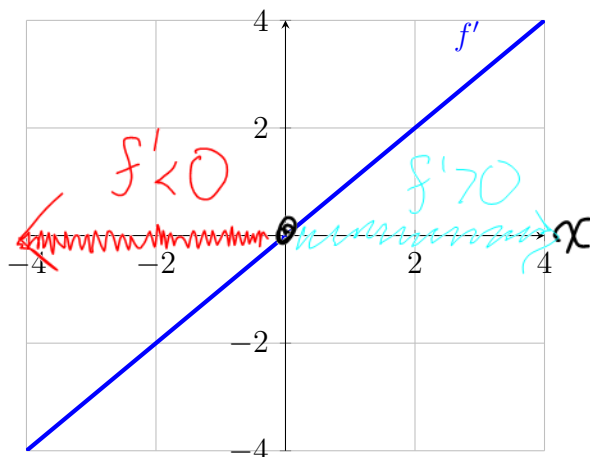


Lecture 20: Graphical Interpretation of the Derivative

Goal: Recover information about a function f from the graph of f' .

1 Implementing the I/D and 1st Derivative Test

Problem 1. Use the graph of f' to determine the following about f :



(a) Intervals of increase/decrease

Crit. Nums: $x=0$



Inc: on $(0, \infty)$

Dec: on $(-\infty, 0)$

(b) Positions of relative max/mins

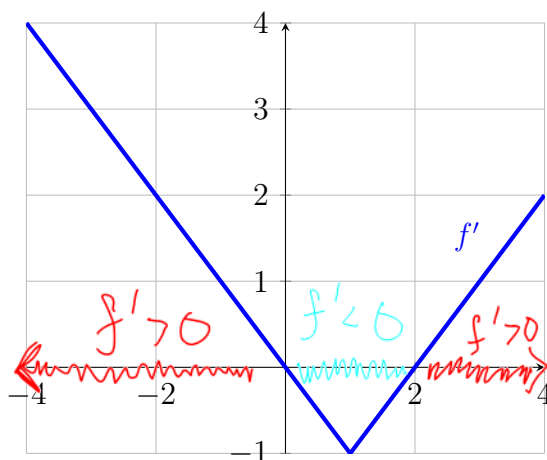
Rel Min: $x=0$

Rel Max: None

by the 1st Derivative Test

Crit. Nums $\leftarrow f'=0 \leftarrow$ when f' touches the x -axis
 $f' < 0 \leftarrow f'$ is below the x -axis
 $f' > 0 \leftarrow f'$ is above the x -axis

Problem 2. Use the graph of f' to determine the following about f :



(a) Intervals of increase/decrease

Crit Nums: $0, 2$



Inc: $(-\infty, 0) \cup (2, \infty)$ Dec: $(0, 2)$

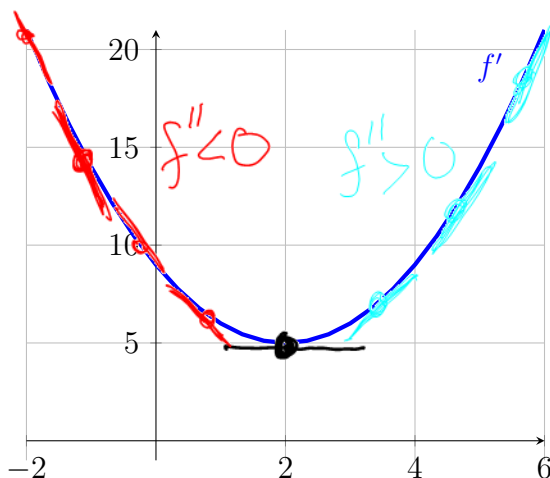
(b) Positions of relative max/mins

Rel Max at $x=0$

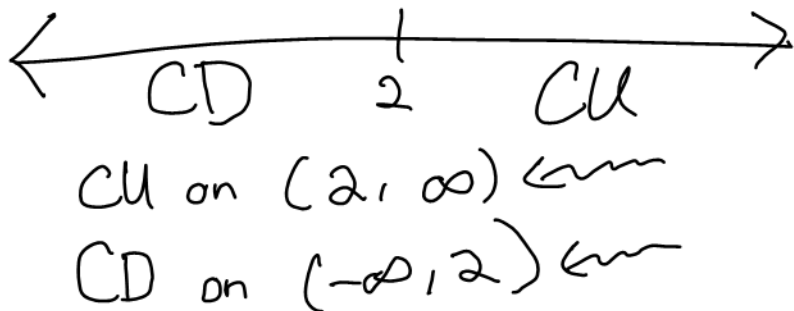
Rel min at $x=2$

2 Implementing the Concavity and 2nd Derivative Test

Problem 3. Use the graph of f' to determine the following about f :



(a) When f is concave up VS. concave down



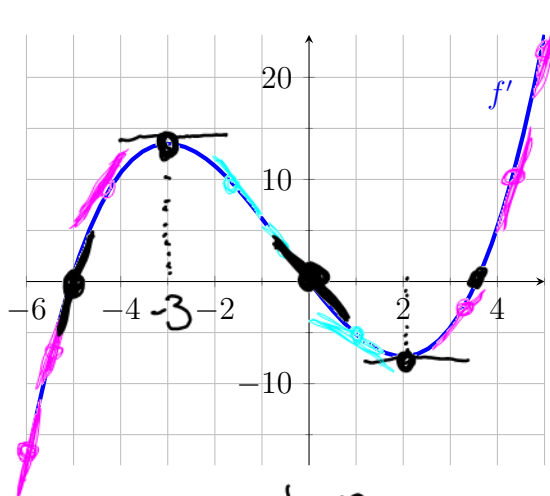
(b) Position of any inflection points

There is an IP
at $x=2$

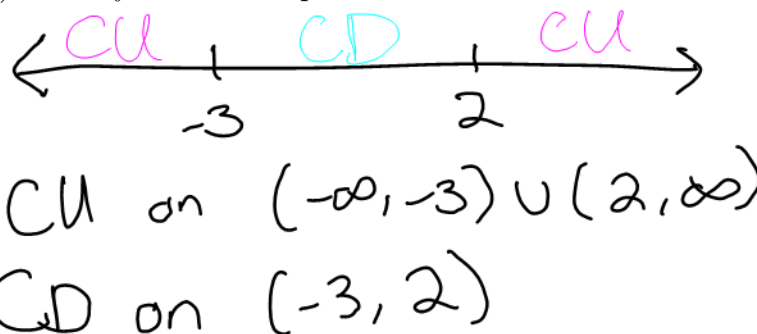
$f''=0 \leftarrow [f']'=0 \leftarrow$ When the slope of tangent lines to f' flat line

$f'' < 0 \leftarrow$ Slope of tangent lines are negative
 $\leftarrow$ The line "point down"

Problem 4. Use the graph of f' to determine the following about f :



(a) When f is concave up VS. concave down



(b) Position of any inflection points

IPs at $x=-3, 2$

(c) Implement the 2nd Derivative Test

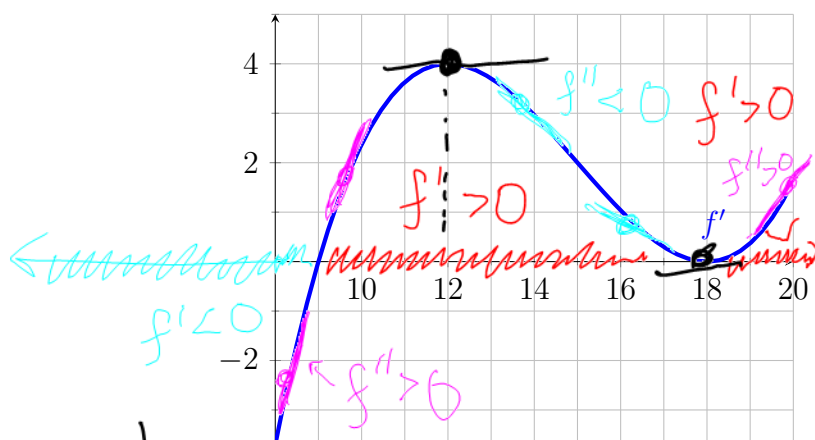
Crit Num: $x=-5, 0, 3+\frac{1}{2}$

$f''(-5) > 0 \cup \Rightarrow$ Rel. Min at $x=5$

$f''(0) < 0 \cap \Rightarrow$ Rel max at $x=0$

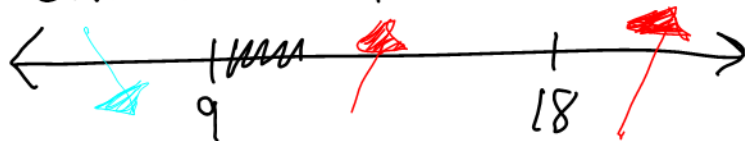
3 Putting Everything Together

Problem 5. Use the graph of f' to determine the following about f :



(a) Intervals of Increase/Decrease

Crit. Nums: $x = 9, 18$



(b) When f is concave up VS concave down



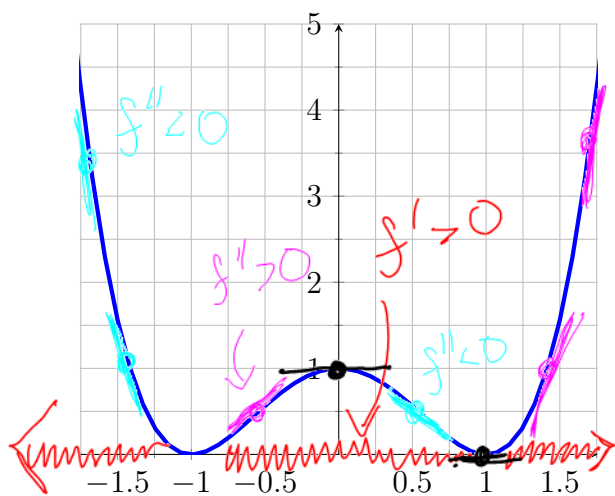
(c) State the locations of any relative max/mins as well as inflection points

Rel Max: None

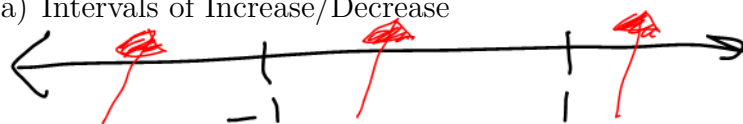
Rel Min: $x = 9$

IPs: $x = 12, 18$

Problem 6. Use the graph of f' to determine the following about f :



(a) Intervals of Increase/Decrease



Inc on $(-\infty, \infty)$

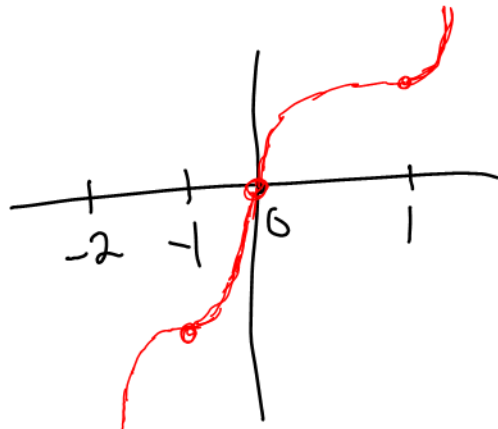
(b) When f is concave up VS concave down



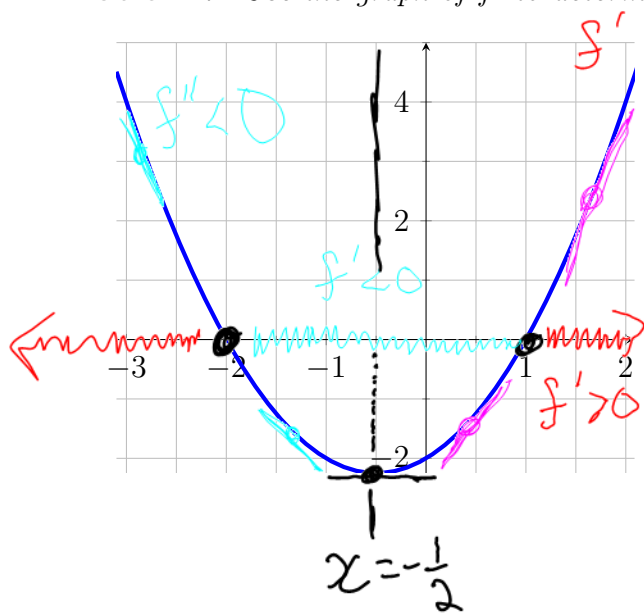
(c) State the locations of any relative max/mins as well as inflection points

Rel Max / Mins: None

IPs: $x = -1, 0, 1$



Problem 7. Use the graph of f' to determine the following about f :

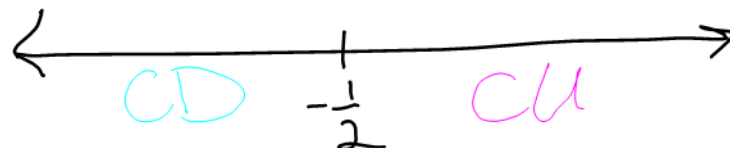


(a) Intervals of Increase/Decrease

Crit Num: $x = -2, 1$



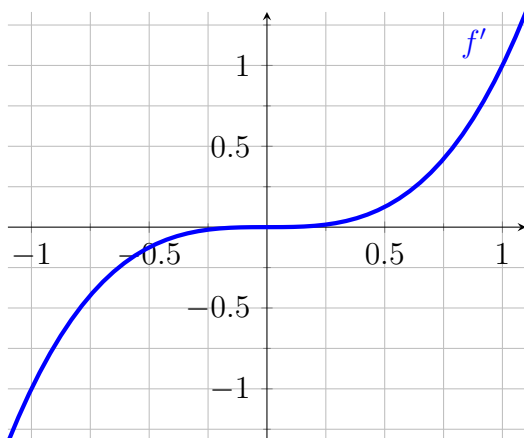
(b) When f is concave up VS concave down



(c) State the locations of any relative max/mins as well as inflection points

Rel Max: $x = -2$ | IP at $x = -\frac{1}{2}$
 Rel Min: $x = 1$

Problem 8. Use the graph of f' to determine the following about f :

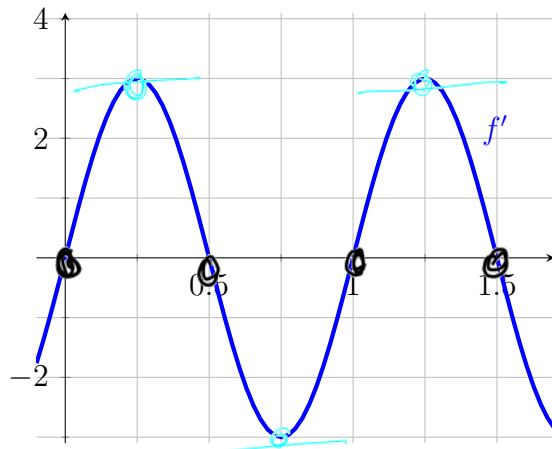


(a) Intervals of Increase/Decrease

(b) When f is concave up VS concave down

(c) State the locations of any relative max/mins as well as inflection points

Problem 9. Use the graph of f' to determine the following about f :



(a) Intervals of Increase/Decrease

(b) When f is concave up VS concave down

(c) State the locations of any relative max/mins as well as inflection points