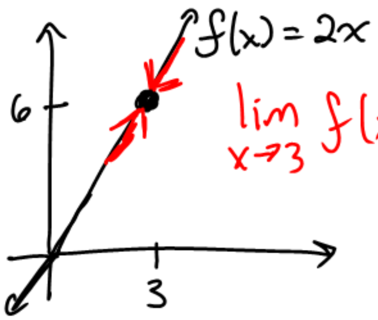
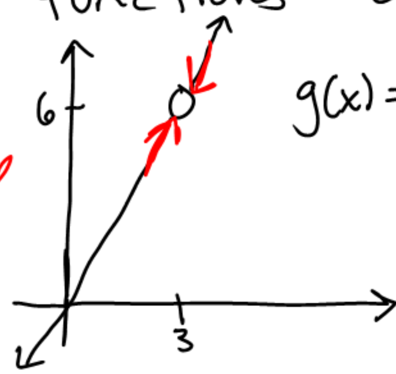
GOAL: Gain an intuitive understanding of what it means for one value to approach another.

Compare these two functions at $x=3$



$\lim\limits_{x\to 3} f(x)=6$



$g(x)=2x\dfrac{(x-3)}{(x-3)}$

$\lim\limits_{x\to 3} g(x)=6$

✱ You will need a calculator today

We say x approaches a (written $x\to a$) to mean "the value of x gets really really close to a."

As $x\to 3$, $f(x)\to 6$, $g(x)\to 6$ even though $g(3)$ is undefined

<u>Def</u>

We write $\lim\limits_{x\to a} f(x)=L$ as shorthand for "as $x\to a$, $f(x)\to L$."

<u>Ex1</u>

Find a plausible value for $\lim\limits_{x\to 4}(2\sqrt{x}-1)=3$

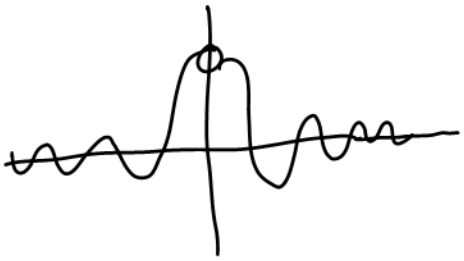
x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$	2.9497	2.995	2.9995		3.005	3.005	3.0497

$2\sqrt{4.1}-1$: $\boxed{4.1}\to\boxed{\sqrt{x}}\to\boxed{X}\to\boxed{2}\to\boxed{=}\to\boxed{-}\to\boxed{1}\to\boxed{=}$

<u>Ex 2</u> Repeat for $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	0.9983	0.99998	0.99999		0.99999	0.9998	0.9983

$\dfrac{\sin(-0.1)}{-0.1}$: $\boxed{0.1} \to \boxed{+/-} \to \boxed{\sin} \to \boxed{\div} \to \boxed{0.1} \to \boxed{+/-} \to \boxed{=}$



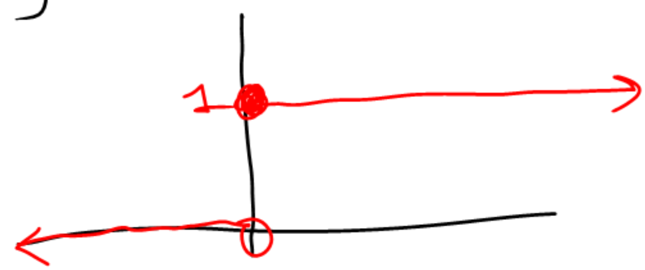
The following limits will need to be memorized

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \ ; \ \lim_{x\to 0} \frac{\cos x - 1}{x} = 0 \ ; \ \lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

<u>Q</u> Can we always find a limit?

<u>A</u> No, consider the following function

$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$



Try to find $\lim_{x\to 0} H(x)$

X	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$H(x)$	0	0	0		1	1	1

$H(x)$ will not approach a particular value, so we say $\lim_{x\to 0} H(x)$ does not exist (DNE)

<u>Q</u> Can we be more specific as to why it DNE

<u>One-Sided Limits</u>

<u>Def</u> The left-sided limit is the value where $f(x) \to L$

as $x \to a$ from the left

$$\lim_{x \to a^-} f(x)$$

$$\lim_{x\to 0^-} H(x) = 0$$

We can introduce a right limit

$$\lim_{x \to a^+} f(x)$$

$$\lim_{x\to 0^+} H(x) = 1$$

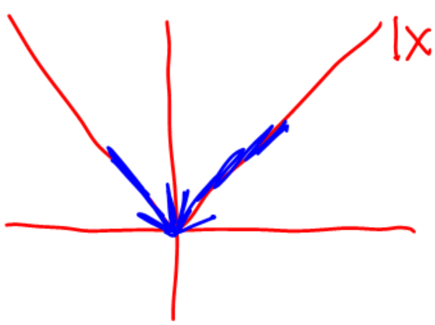
Remark

① If these are unequal, then $\lim_{x\to a} f(x)$ DNE

② If they are equal, then $\lim_{x\to a} f(x) = \lim_{x\to a^-} f(x)$

$$= \lim_{x\to a^+} f(x)$$

Ex3 Find $\lim_{x\to 0} |x|$



$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\lim_{x\to 0^-} |x| = \lim_{x\to 0^-} (-x) = 0$$

$$\lim_{x\to 0^+} |x| = \lim_{x\to 0^+} x = 0$$

So $\lim_{x\to 0} |x| = x$

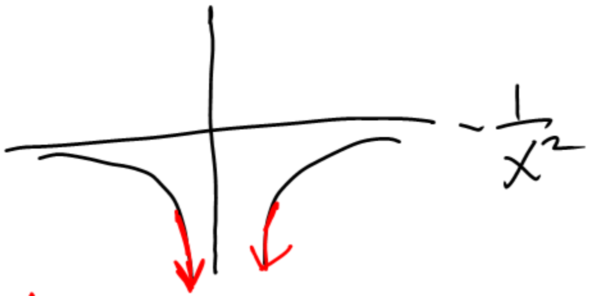
Infinite Limit

$$f(x) = \frac{1}{x^2}$$

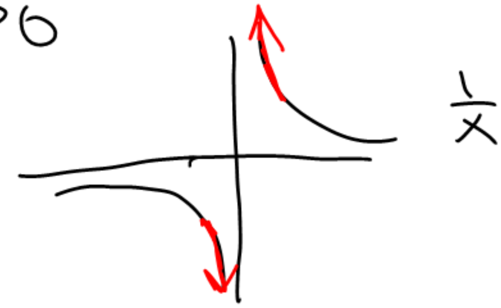
x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$\frac{1}{x^2}$	100	10^4	10^6		10^6	10^4	100

$$\left(\frac{1}{0.1}\right)^2 = \frac{1}{\frac{1}{100}} = 100$$

We say $\lim_{x\to 0} \frac{1}{x^2} = \infty$, ie, $\frac{1}{x^2}$ grows unbounded as $x \to 0$

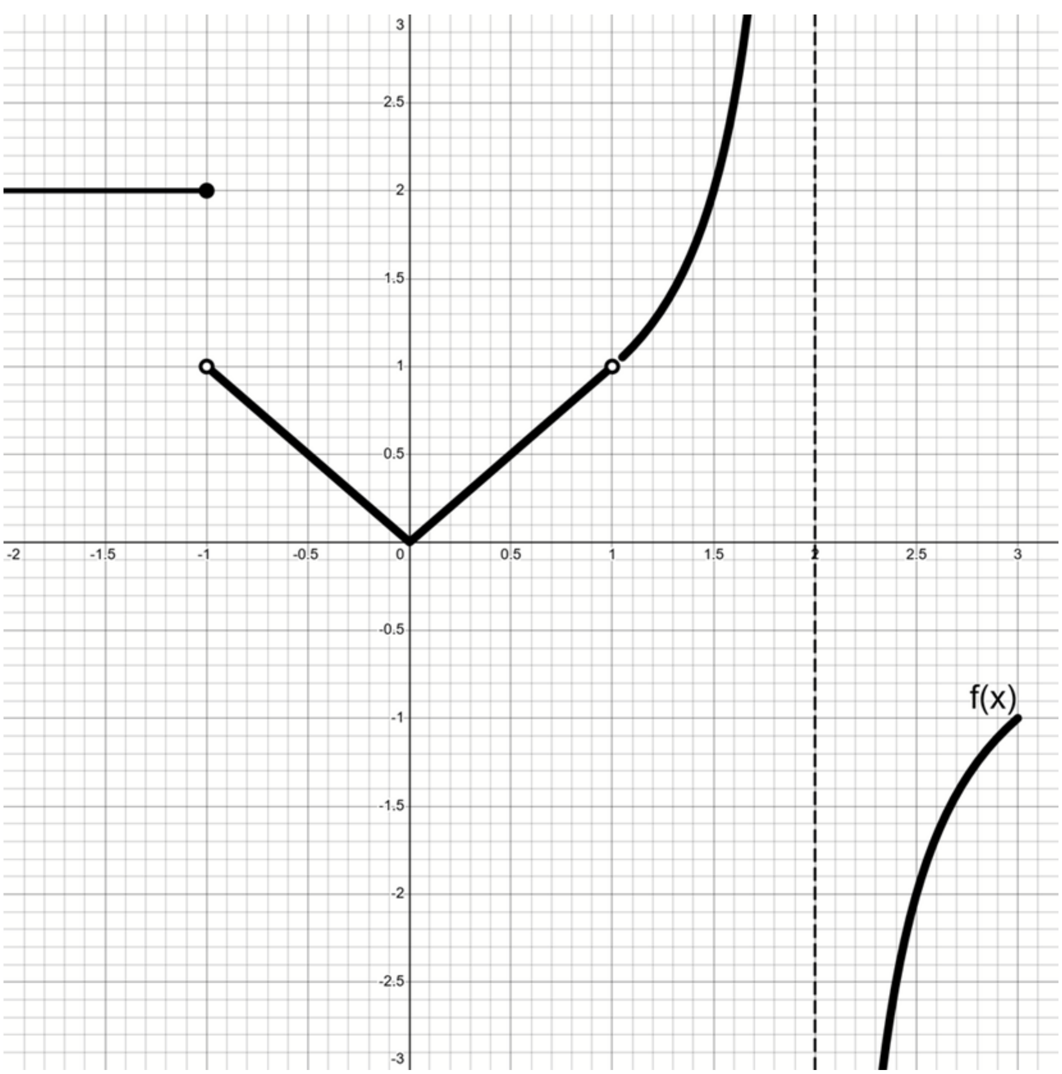


$\lim_{x\to 0}\left(-\frac{1}{x^2}\right) = -\infty$



$\lim_{x\to 0} \frac{1}{x}$ DNE

<u>NOTE</u> If the limit equals ∞ or $-\infty$, technically the limit DNE. <u>BUT</u> we are being more specific as to why it DNE


f(x)
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
-0.5
-1
-1.5
-2
-2.5
-3
0.5
1
1.5
2
2.5
3