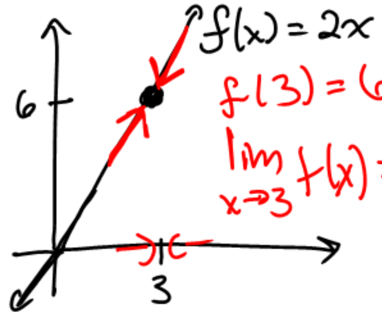
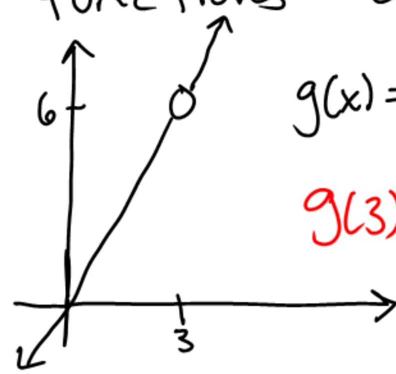
GOAL: Gain an intuitive understanding of what it means for one value to approach another.

Compare these two functions at $x = 3$



$f(3) = 6$

$\lim_{x \to 3} f(x) = 6$



$g(x) = 2x \frac{(x-3)}{(x-3)}$

$g(3)$ undefined

$\lim_{x \to 3} g(x) = 6$

✱ You will need a calculator today

We say <u>x approaches a</u> (written $x \to a$) to mean "x gets really really close to a."

As $x \to 3$, $f(x) \to 6$, $g(x) \to 6$

<u>Def</u> We write $\lim_{x \to a} f(x) = L$ as shorthand for "as $x \to a$, $f(x) \to L$." L is called the <u>limit</u> of $f(x)$ as $x \to a$.

<u>Ex1</u> Find a plausible value for $\lim_{x \to 4} (2\sqrt{x} - 1) = 3$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$	2.9497	2.995	2.9995		3.0005	3.005	3.0497

$2\sqrt{4.1} - 1$: $\boxed{4.1} \to \boxed{\sqrt{x}} \to \boxed{\times} \to \boxed{2} \to \boxed{=} \to \boxed{-} \to \boxed{1} \to \boxed{=}$

<u>Ex2</u> Repeat for $\lim_{x \to 0} \frac{\sin x}{x} = 1$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	0.9983	0.9999	0.99999		0.99999	0.9999	0.9983

$\dfrac{\sin(-0.1)}{-0.1}$ $\boxed{0.1} \to \boxed{+/-} \to \boxed{\text{SIN}} \to \boxed{\div} \to \boxed{0.1} \to \boxed{+/-} \to \boxed{=}$

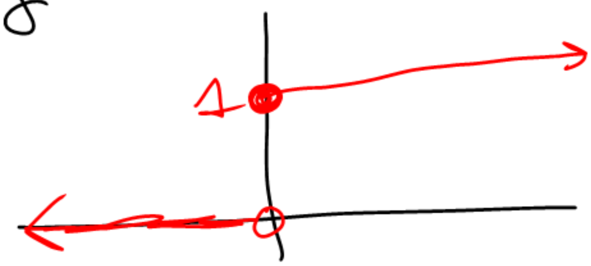
The following will need to be memorized

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \; ; \; \lim_{x\to 0} \frac{\cos x - 1}{x} = 0 ; \; \lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

<u>Q</u> Can we always find a limit?

<u>A</u> No, consider the following

$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$



x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$H(x)$	0	0	0		1	1	1

H(x) will not approach a particular value, so we say $\lim_{x\to 0} H(x)$ does not exist (DNE)

<u>Q</u> Can we be more specific as to why it DNE

One-Sided Limit

We write $\lim_{x\to a^-} f(x) = L$ to mean "$f(x) \to L$ as $x \to a$ from the left."

$$\lim_{x\to 0^-} H(x) = 0$$

Similarly, $\lim_{x\to a^+} f(x) = L$ means "$f(x) \to L$ as $x \to a$ from the right."

$$\lim_{x\to 0^+} H(x) = 1$$

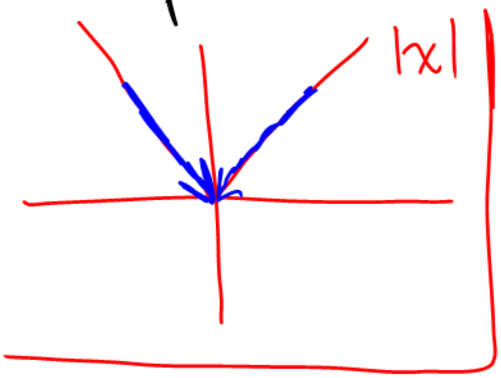
Remark ① If these are unequal, then the limit DNE

② If they are equal, then

$$\lim_{x\to a} f(x) = \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$$

Ex 4

Compute $\lim_{x\to 0} |x|$

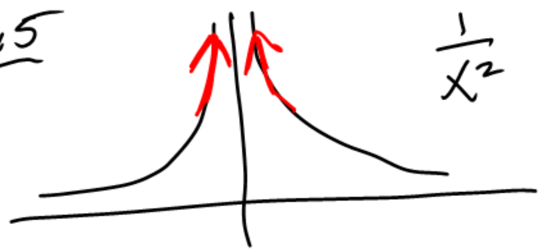


$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\lim_{x\to 0^-} |x| = \lim_{x\to 0^-} (-x) = 0$$
$$\lim_{x\to 0^+} |x| = \lim_{x\to 0^+} x = 0$$
$$\Longrightarrow \lim_{x\to 0} |x| = 0$$

Infinite Limit

Ex5

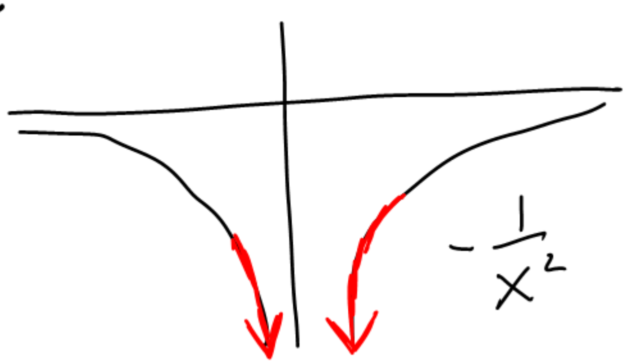


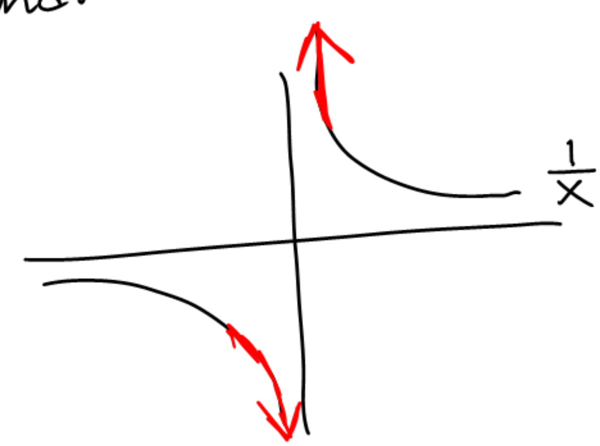
Compute $\lim_{x\to 0} \frac{1}{x^2}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	100	10^4	10^6		10^6	10^4	100

We say $\lim_{x\to 0} \frac{1}{x^2} = \infty$. That is, as $x \to 0$, $f(x)$ keeps increasing w/o bound.

Ex6



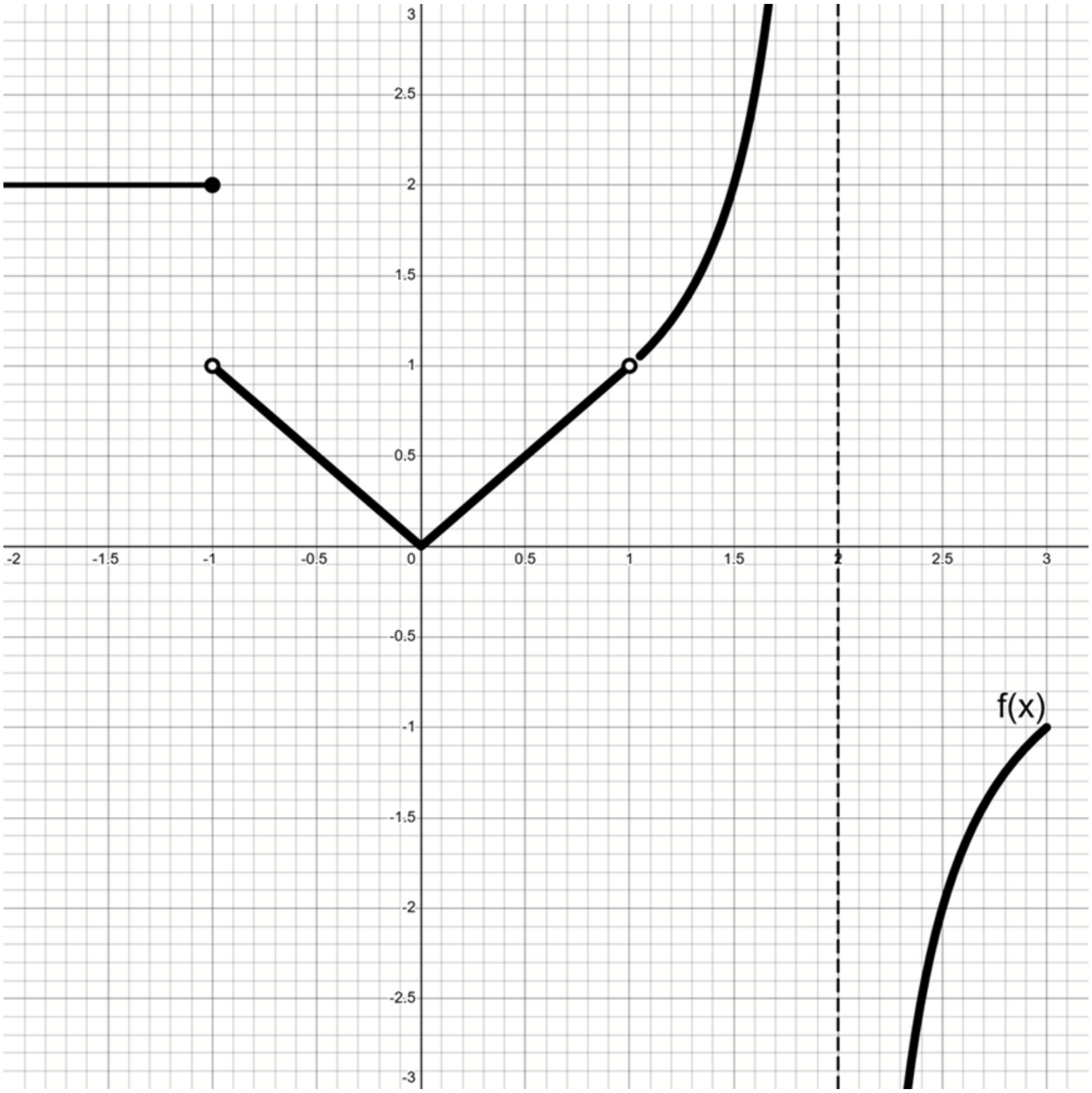


$$\lim_{x\to 0}\left(-\frac{1}{x^2}\right) = -\infty$$

$\lim_{x\to 0}\frac{1}{x}$ DNE

<u>Remark</u>

If the limit is ∞ or $-\infty$, technically the limit DNE. <u>BUT</u>, we are being more specific as to why it DNE.


f(x)