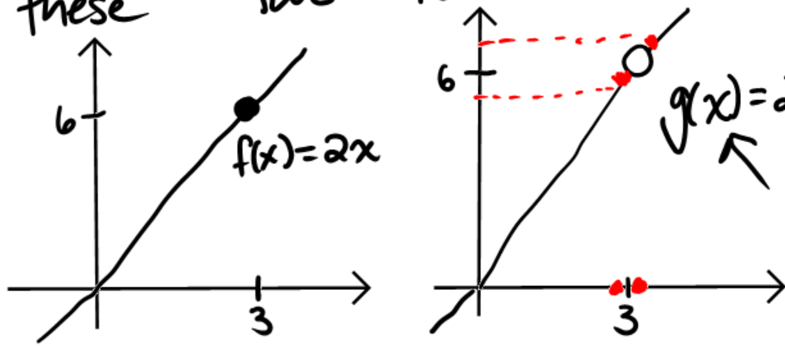
# Lecture 2: Intro to Limits

Compare these two functions at $x=3$




$g(x) = 2x\dfrac{(x-3)}{(x-3)}$

$\lim\limits_{x\to 3} g(x) = 6$

⚠ You will need a calculator today

We say $x$ approaches $a$ (written $x \to a$) when we mean "the value of $x$ gets really really close to $a$."

As $x \to 3$, $f(x) \to 6$, $g(x) \to 6$

**Def** We write $\lim\limits_{x\to a} f(x) = L$ as shorthand for "as $x \to a$, then $f(x) \to L$."

$L$ is the limit of $f$ as $x \to a$

Ex/ Find a plausible value for

$$\lim_{x\to 4} (2\sqrt{x} - 1)$$

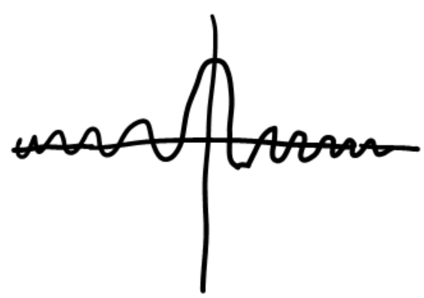
NOTE: Parantheses are need when there are 2+ terms

| $x$ | 3.9 | 3.99 | 3.999 | 4 | 4.001 | 4.01 | 4.1 |
|---|---|---|---|---|---|---|---|
| $2\sqrt{x}-1$ | 2.9497 | 2.995 | 2.9995 | | 3.0005 | 3.005 | 3.0497 |

$\lim\limits_{x\to 4} (2\sqrt{x} - 1) = 3$

Ex/ Find a plausible for $\lim\limits_{x\to 0} \dfrac{\sin x}{x}$

| $x$ | -0.1 | -0.01 | -0.001 | 0 | 0.001 | 0.01 | 0.1 |
|---|---|---|---|---|---|---|---|
| $\frac{\sin x}{x}$ | 0.9983 | 0.9998 | 0.9999 | | 0.9999 | 0.9998 | 0.9983 |



$\boxed{0.001}$ $\boxed{\sin}$ $\boxed{/}$ $\boxed{0.001}$ $\boxed{\text{ENTER}}$

$\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$

Limits you will need to remember

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \;;\; \lim_{x\to 0}\frac{\cos x-1}{x}=0 \;;\; \lim_{x\to 0}\frac{e^x-1}{x}=1$$

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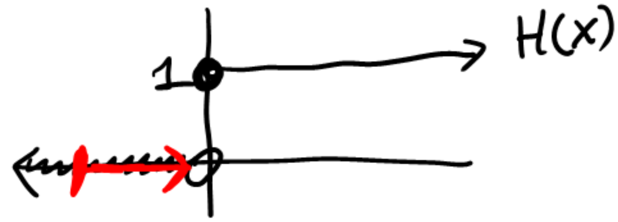
## One-Sided Limits

Q: Can we always find a limit?

A: No! Consider the Heaviside Function:

$$H(x)=\begin{cases}1 & x\geq 0\\ 0 & x<0\end{cases}$$




| $x$ | -0.001 | -0.01 | -0.1 | 0 | 0.001 | 0.01 | 0.1 |
|---|---|---|---|---|---|---|---|
| $H(x)$ | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

As $x\to 0$, $H(x)$ does not approaches a value, in this case $\lim_{x\to 0} H(x)$ does not exists (abbreviated DNE)

Def The left-sided limit is the value ($L$) $f(x)$ approachs as $x\to a$ from the left

$$\lim_{x\to a^-} f(x)=L \;;\; \text{Ex/ } \lim_{x\to 0^-} H(x)=0$$

Similiarly, $\lim_{x\to a^+} f(x)=L$ means "$f(x)\to L$ as $x\to a$ from the right"

Ex/ $\lim_{x\to 0^+} H(x)=1$

Remark ① If $\lim_{x\to a^+} f(x)=\lim_{x\to a^-} f(x)=L$, then $\lim_{x\to a} f(x)=L$

② If $\lim_{x\to a^+} f(x)\neq\lim_{x\to a^-} f(x)$, then $\lim_{x\to a} f(x)$ DNE

Ex of ①

$f(x)=|x|$

$$\lim_{x\to 0^-}|x|=\lim_{x\to 0^-}(-x)=0$$

$$\lim_{x\to 0^+}|x|=\lim_{x\to 0^+}x=0$$

# Infinite Limits

Ex/ $f(x) = \frac{1}{x^2}$

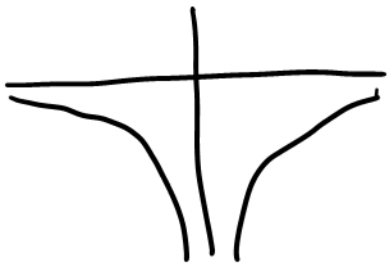
Let's compute values around 0

| $x$ | $-0.1$ | $-0.01$ | $-0.001$ | $0$ | $0.001$ | $0.01$ | $0.1$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $10^2$ | $10^4$ | $10^6$ | | $1\,000\,000$ | $10000$ | $100$ |

$\frac{1}{(\frac{1}{10})^2} = \frac{1}{\frac{1}{100}} = 100$ ; $\frac{1}{(\frac{1}{100})^2} = 10\,000$ ; $\frac{1}{(\frac{1}{1000})^2} = 10^6$

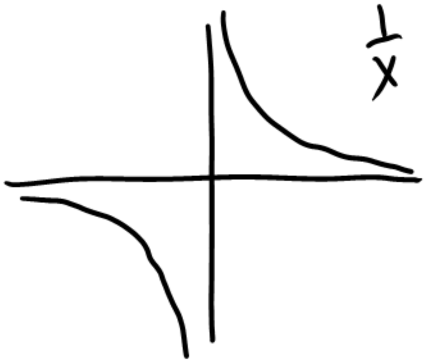
We say $\lim_{x\to 0} \frac{1}{x^2} = \infty$. Ie, $\frac{1}{x^2}$ is unbounded as $x \to 0$

Ex/ $f(x) = \frac{-1}{x^2}$



$\lim_{x\to 0} \frac{-1}{x^2} = -\infty$. Ie, $\frac{-1}{x^2}$ decreases without bound as $x \to 0$.

Ex/ $f(x) = \frac{1}{x}$

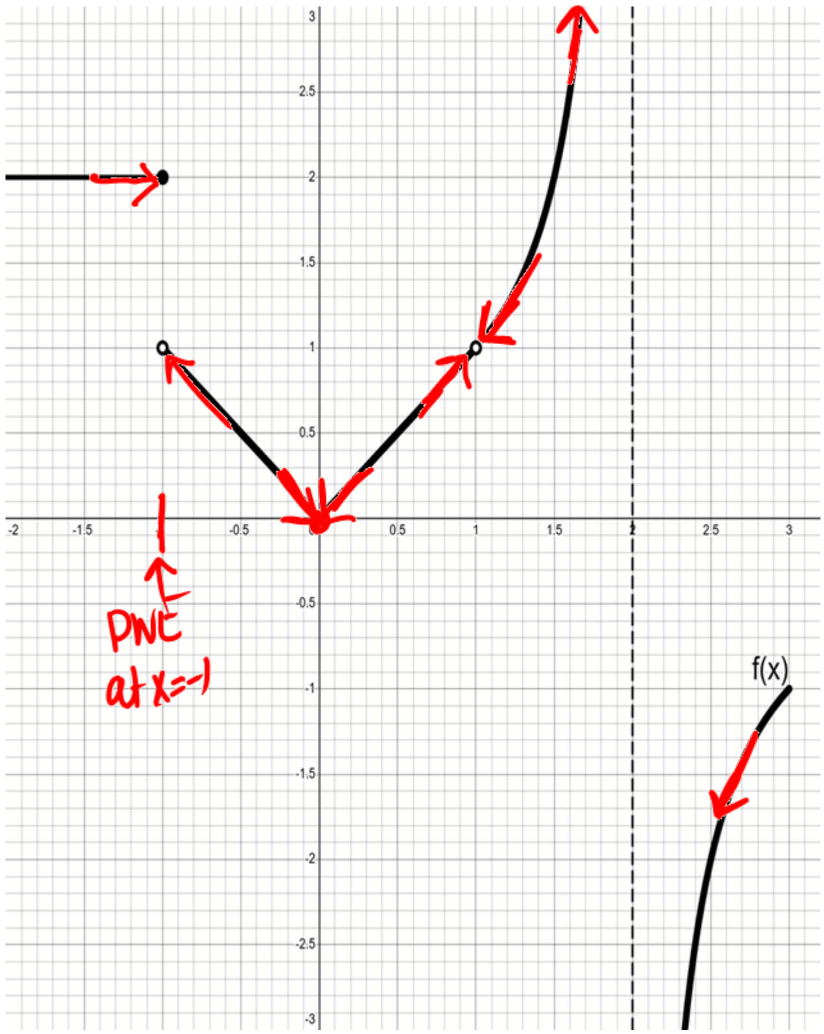



$\lim_{x\to 0^+} \frac{1}{x} = \infty$

$\lim_{x\to 0^-} \frac{1}{x} = -\infty$

$\lim_{x\to 0} \frac{1}{x}$ DNE

NOTE: If $\lim_{x\to a} f(x) = \infty$ or $-\infty$, technically $\lim_{x\to a} f(x)$ DNE. But, we are being more specific as to why it DNE




# Computing Limits Graphically

$\lim_{x\to -1^-} f(x) = 2$

$\lim_{x\to -1^+} f(x) = 1$

$\lim_{x\to 0} f(x) = 0$

$\lim_{x\to 1} f(x) = 1$

$\lim_{x\to 2^-} f(x) = \infty$

$\lim_{x\to 2^+} f(x) = -\infty$