$f(x) = \frac{9}{8x^2}$, find $f'(x)$ HW Problem

$\frac{9}{8} x^{-2}$

$\frac{-18}{8} x^{-3}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{9}{8(x+\Delta x)^2} - \frac{9}{8x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{72x^2 - 9 \cdot 8(x+\Delta x)^2}{64x^2(x+\Delta x)^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{72x^2 - 72(x^2 + 2x\Delta x + (\Delta x)^2)}{64x^2(x+\Delta x)^2} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-72(2x\Delta x) - 72(\Delta x)^2}{64x^2(x+\Delta x)^2} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-72(2)x - 72\Delta x}{64x^2(x+\Delta x)^2} = \frac{-144x}{64x^4} = \frac{-144}{64x^3}$$

$$= \boxed{-\frac{9}{8} \cdot \frac{2}{x^3}} = \frac{-18}{8x^3}$$

Lecture 6 - Basic Derivative Rules

Goal Find derivatives w/o relying on def

Derivative of Polynomial

Function	Degree	Derivative
constant	0	0
x	1	$1 = 1 \cdot x^{1-1}$
x^2	2	$2x = 2 \cdot x^{2-1}$
x^3	3	$3x^2 = 3 \cdot x^{3-1}$
x^4	4	$4x^3 = 4 \cdot x^{4-1}$

$F(x) = c$

$F'(x) = \lim_{\Delta x \to 0} \frac{c - c}{\Delta x} = 0$

$F(x) = x$

$F'(x) = \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x} = 1$

Theorem (Power Rule) Let p be any real number. Then

$$\frac{d}{dx}\left(x^p\right) = p\,x^{p-1} = p x^{p-1}$$

Proof Postponed to Lecture 11 (Note: $x^p = e^{p\ln(x)}$)

Ex/ Compute

① $\frac{d}{dx}\left(x^{10}\right) = 10 \cdot x^{10-1} = 10x^9$

$p = 10$

② $\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}\left(x^{-2}\right) = -2 \cdot x^{-2-1} = -2x^{-3} = \frac{-2}{x^3}$

③ $\frac{d}{dx}\left(\sqrt[3]{x}\right) = \frac{d}{dx}\left(x^{\frac{1}{3}}\right) = \frac{1}{3} x^{\frac{1}{3}-1} = \underbrace{\frac{1}{3} x^{-\frac{2}{3}}} = \frac{1}{3} \cdot \frac{1}{x^{\frac{2}{3}}}$

$= \frac{1}{3\sqrt[3]{x^2}}$

Q: How can we differentiate $x^2 - 2x + 3$?

Theorem (Linearity) Let f and g be differentiable functions

① $\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$ [Sum/Difference Rule]

② For any constant c, $\frac{d}{dx}\left(cf(x)\right) = c \cdot \frac{d}{dx}\left(f(x)\right)$

[Constant Multiple Rule]

Ex/ Compute $\frac{d}{dx}(x^2 - 2x + 3)$

$$\frac{d}{dx}(x^2 - 2x + 3) = \frac{d}{dx}(x^2) - \frac{d}{dx}(2x) + \frac{d}{dx}(3)$$

$$= \frac{d}{dx}(x^2) - 2 \cdot \frac{d}{dx}(x) + \frac{d}{dx}(3)$$

$\frac{d}{dx}(x^2) = 2x^{2-1}$

$\frac{d}{dx}(x) = 1 \cdot x^{1-1}$

$$= 2x - 2 \cdot 1 + 0$$

$$= 2x - 2$$

Ex/ Find $\frac{d}{dx}\left[\frac{(2x)^\pi}{\sqrt{x}}\right] = \frac{d}{dx}\left[\frac{2^\pi \cdot x^\pi}{\sqrt{x}}\right] = \frac{d}{dx}\left[\frac{2^\pi \cdot x^\pi}{x^{\frac{1}{2}}}\right]$

$$= \frac{d}{dx}\left[2^\pi \cdot x^{\pi - \frac{1}{2}}\right] = 2^\pi \frac{d}{dx}\left(x^{\pi - \frac{1}{2}}\right) = 2^\pi \cdot \left(\pi - \tfrac{1}{2}\right) x^{\pi - \frac{1}{2} - 1}$$

$$= 2^\pi \left(\pi - \tfrac{1}{2}\right) x^{\pi - \frac{3}{2}}$$

Derivatives of Sine and Cosine

Theorem

① $\frac{d}{dx}(\sin x) = \cos x$

② $\frac{d}{dx}(\cos x) = -\sin x$

Negative sign. Don't forget it.

Proof ① See your textbook

Recall $\cos(a+b) = \cos a \cos b - \sin a \sin b$

② $$\frac{d}{dx}(\cos x) = \lim_{\Delta x \to 0} \frac{\cos(x + \Delta x) - \cos x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \cos x \underbrace{\left[\frac{\cos \Delta x - 1}{\Delta x}\right]}_{0} - \lim_{\Delta x \to 0} \sin x \underbrace{\left[\frac{\sin \Delta x}{\Delta x}\right]}_{1}$$

$$= -\sin x$$

Ex/ Find y' if $y = 2\sin x - 5\cos x$

$$y' = 2(\sin x)' - 5(\cos x)' = 2\cos x - 5(-\sin x)$$
$$= 2\cos x + 5\sin x$$

Ex/ Find the equation of the tangent line for $f(x) = 3\sin x$ at $x = \pi$

Slope : $f'(\pi)$. $f'(x) = (3\sin x)' = 3(\sin x)' = 3\cos x$

$$f'(\pi) = 3\cos\pi = 3(-1) = -3$$

Point : $(\pi, f(\pi)) = (\pi, 3\sin\pi) = (\pi, 0)$

Equation:
$$y - 0 = -3(x - \pi)$$
$$y = -3x + 3\pi$$

$e^a e^b = e^{a+b}$

Derivative of e^x

$$\frac{d}{dx}(e^x) = \lim_{\Delta x \to 0} \frac{e^{x+\Delta x} - e^x}{\Delta x} = \lim_{\Delta x \to 0} \frac{e^x e^{\Delta x} - e^x}{\Delta x}$$

$$= e^x \lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x} = e^x \cdot 1$$

Theorem $y = e^x$ is the only function that satisfies the following Initial Value Problem (IVP)

$$\begin{cases} \frac{dy}{dx} = y \leftarrow \frac{d}{dx}(e^x) = e^x \\ y(0) = 1 \leftarrow e^0 = 1 \end{cases}$$

Ex/ What is $y'(0)$ if $y = 5e^x + 7$

$y' = (5e^x + 7)' = 5 \cdot (e^x)' + (7)' = 5e^x$

$y'(0) = 5 \cdot e^0 = 5 \cdot 1 = 5$

Ex/ When does $y' = 1$ if $y = 72e^x$

$y' = (72e^x)' = 72 \cdot (e^x)' = 72e^x \underline{\underline{\text{set}}}\ 1$

$72e^x = 1$

$\ln(e^x) = \ln\left(\frac{1}{72}\right)$

$x = \ln\left(\frac{1}{72}\right) = \ln\left(72^{-1}\right) = -\ln(72)$

Summary: Power Rule: $\frac{d}{dx}(x^p) = px^{p-1}$ $\qquad \frac{d}{dx}(e^x) = e^x$

$(\sin x)'$ $\qquad \frac{d}{dx}(\sin x) = \cos x$

$(\cos x)'$ $\qquad \frac{d}{dx}(\cos x) = -\sin x$