$f(x) = \frac{9}{8x^2}$, find $f'(x)$ HW Problem

$\frac{9}{8} x^{-2}$

$\frac{-18}{8} x^{-3}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{9}{8(x+\Delta x)^2} - \frac{9}{8x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{72x^2 - 9 \cdot 8(x+\Delta x)^2}{64x^2(x+\Delta x)^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{72x^2 - 72(x^2 + 2x\Delta x + (\Delta x)^2)}{64x^2(x+\Delta x)^2} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-72(2x\Delta x) - 72(\Delta x)^2}{64x^2(x+\Delta x)^2} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-72(2)x - 72\Delta x}{64x^2(x+\Delta x)^2} = \frac{-144x}{64x^4} = \frac{-144}{64x^3}$$

$$= \boxed{-\frac{9}{8} \cdot \frac{2}{x^3}} = \frac{-18}{8x^3}$$

Lecture 6 Basic Derivative Rules

Goal Compute derivatives w/o relying on def

Derivative of Polynomials

Function	Degree	Derivative
Constant	0	0
x	1	$1 = 1\cdot x^0 = 1\cdot x^{1-1}$
x^2	2	$2x = 2\cdot x^{2-1}$
x^3	3	$3x^2 = 3\cdot x^{3-1}$
x^4	4	$4x^3 = 4\cdot x^{4-1}$

$F(x) = c$

$F'(x) = \lim_{\Delta x\to 0} \frac{c-c}{\Delta x} = 0$

$F(x) = x$

$F'(x) = \lim_{\Delta x \to 0} \frac{x+\Delta x - x}{\Delta x} = 1$

Theorem (Power Rule) Let p be any real number. Then

$$\frac{d}{dx}\left(x^p\right) = p\cdot x^{p-1} = p x^{p-1}$$

Proof Postpone to Lecture 11 (Note: $x^p = e^{p\ln(x)}$)

Ex/ Compute

① $\frac{d}{dx}\left(x^{10}\right) = 10\cdot x^{10-1} = 10x^9$ $p=10$

② $\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}\left(x^{-2}\right) = -2\cdot x^{-2-1} = -2x^{-3} = \frac{-2}{x^3}$ $p=-2$

③ $\frac{d}{dx}\left(\sqrt[3]{x}\right) = \frac{d}{dx}\left(x^{\frac{1}{3}}\right) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}\cdot x^{-\frac{2}{3}} = \frac{1}{3x^{2/3}}$ $p=\frac{1}{3}$

$= \frac{1}{3\sqrt[3]{x^2}}$

Q: How can we differentiate a function like

$$x^2 - 2x + 3 \quad ?$$

Theorem (Linearity) Let f and g be differentiable functions:

① $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$ [Sum/Difference Rule]

② For a constant c, $\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}(f(x))$ [Constant Multiple Rule]

Ex/ What's $\frac{d}{dx}(x^2-2x+3)$?

$$\frac{d}{dx}(x^2-2x+3) = \frac{d}{dx}(x^2) - \frac{d}{dx}(2x) + \frac{d}{dx}(3)$$
$$= \frac{d}{dx}(x^2) - 2\frac{d}{dx}(x) + \frac{d}{dx}(3)$$
$$= 2x - 2\cdot 1 + 0 = \boxed{2x-2}$$

Ex/ $$\frac{d}{dx}\left[\frac{(2x)^\pi}{\sqrt{x}}\right] = \frac{d}{dx}\left[\frac{2^\pi \cdot x^\pi}{\sqrt{x}}\right] = \frac{d}{dx}\left[\frac{2^\pi \cdot x^\pi}{x^{\frac{1}{2}}}\right] = \frac{d}{dx}\left[2^\pi x^{\pi-\frac{1}{2}}\right]$$
$$= 2^\pi \frac{d}{dx}\left[x^{\underbrace{\pi-\frac{1}{2}}_{p=\pi-\frac{1}{2}}}\right] = 2^\pi \cdot (\pi-\tfrac{1}{2})\, x^{\pi-\frac{1}{2}-1}$$
$$= \boxed{2^\pi(\pi-\tfrac{1}{2})\, x^{\pi-\frac{3}{2}}}$$

Derivatives of Sine and Cosine

Theorem ① $\frac{d}{dx}(\sin x) = \cos x$

② $\frac{d}{dx}(\cos x) = -\sin x$ Negative sign. Don't forget it

Proof ① See Textbook

② $$\frac{d}{dx}(\cos x) = \lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - \cos x}{\Delta x}$$ [Recall $\cos(a+b) = \cos a \cos b - \sin a \sin b$]

$$= \lim_{\Delta x \to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x}$$

$$= \cos x \Bigg[\underbrace{\lim_{\Delta x\to 0} \frac{\cos\Delta x - 1}{\Delta x}}_{0}\Bigg] - \sin x \Bigg[\underbrace{\lim_{\Delta x \to 0} \frac{\sin\Delta x}{\Delta x}}_{1}\Bigg] = -\sin x$$

Ex/ Find y' if $y = 2\sin x - 5\cos x$

$$y' = (2\sin x - 5\cos x)' = 2(\sin x)' - 5(\cos x)' = 2\cos x - 5(-\sin x)$$
$$= 2\cos x + 5\sin x$$

Ex/ Find the equation of the tangent line for $f(x) = 3\sin x$ at $x = \pi$

Slope: $f'(\pi)$. $f'(x) = (3\sin x)' = 3(\sin x)' = 3\cos x$

$$f'(\pi) = 3\cos\pi = -3$$

Point: $(\pi, f(\pi)) = (\pi, 3\sin\pi) = (\pi, 0)$

Equation:
$$y - 0 = -3(x - \pi)$$
$$y = -3x + 3\pi$$

Ex/ Compute y' if $y = 2\cos x + 5\sin x$

$$y' = 2(\cos x)' + 5(\sin x)' = -2\sin x + 5\cos x$$
$$= (-1)(\underbrace{2\sin x - 5\cos x}_{\text{Starting point in the ex. 2 ex.s ago}})$$

Derivative of e^x

$$\frac{d}{dx}(e^x) = \lim_{\Delta x \to 0} \frac{e^{x+\Delta x} - e^x}{\Delta x} = \lim_{\Delta x\to 0} \frac{e^x e^{\Delta x} - e^x}{\Delta x} = e^x \underbrace{\lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x}}_{1}$$
$$= e^x$$

$$e^a e^b = e^{a+b}$$

<u>Theorem</u> $y = e^x$ is the only function that satisfies the following <u>Initial Value Problem</u> (IVP)

$$\begin{cases} \frac{dy}{dx} = y \leftarrow \boxed{\frac{d}{dx}(e^x) = e^x} \\ y(0) = 1 \leftarrow e^0 = 1 \end{cases}$$

Ex/ What's $y'(0)$ if $y = 5e^x + 7$

$$y' = 5[e^x]' + [7]' = 5e^x$$

$$y'(0) = 5 \cdot e^0 = 5 \cdot 1 = 5$$

Ex/ When does $y' = 1$ if $y = 72e^x$

$$y' = 72[e^x]' = 72e^x \underset{=}{\overset{\text{set}}{=}} 1$$

$$72e^x = 1$$

$$\ln(e^x) = \ln\left(\frac{1}{72}\right)$$

$$x = \ln\left(\frac{1}{72}\right) = \ln(72^{-1})$$

$$= -\ln(72)$$

Summary: $\frac{d}{dx}(x^p) = px^{p-1}$ $\qquad$ $\frac{d}{dx}(e^x) = e^x$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$