MT 1 Review

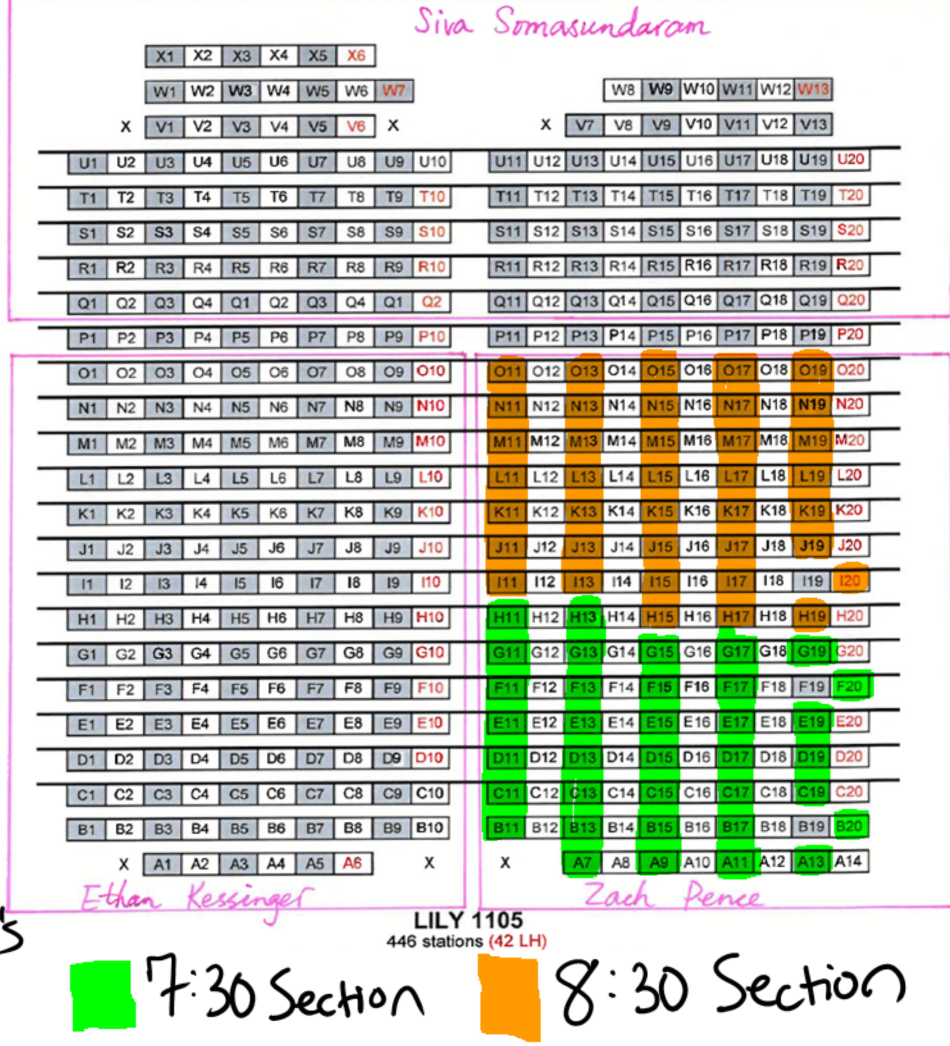
- Tomorrow (Tues. 9/23)
- 8-9PM
- LILY 1105
- Lectures 2-10

Make sure to bring:

- Pencils / Erasers
- PUID
- Calculator

Section Number / Instructor's Name

- 7:30 — Sec. 19
- 8:30 — Sec. 20



This is an optional assignment that will be worth 2 points of extra credit. You must show work to get credit.

NOTE: These are just review problems of Lectures 2-10; these are not necessarily representative of the problems of the exam. The exam can (and most likely will) have different problems.

Directions:

1. Complete each problem on the next page, make sure to show your work. Clearly mark the question number and final answer.
2. You have two options to turn in this assignment:
   (a) **In-person:** You can slip it under my office door located in MATH 615. Make sure your name is on it and that it is stapled together (if there are multiple pages).
   (b) **Email:** You may email your assignment to me at pence11@purdue.edu
      i. Scan your assignment so that it is one PDF (do not submit a bunch of images).
      ii. In the subject line, write "EXTRA CREDIT 1 [your name]".
3. The answers will be given in the lecture after the due date (9/22). Therefore, **no late submissions will be allowed**.

$\sqrt{2}$ $\boxed{2}$ $\boxed{\Gamma}$

Problem 1. *Use the table below to compute numerically* $\lim_{x\to 0} \frac{e^{3x}-1}{x} = 3$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	2.5918	2.9534	2.9955	—	3.0045	3.0455	3.4986

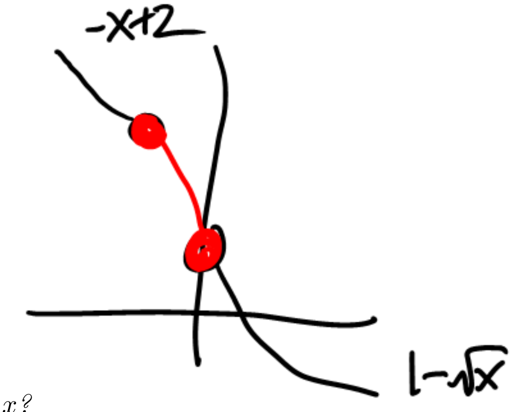
$\boxed{0.001}$ $\boxed{x}$ $\boxed{3}$ $\boxed{=}$ $\underbrace{\boxed{2^{nd}}\ \boxed{LN}}_{e^x}$ $\boxed{\equiv}$ $\boxed{-}$ $\boxed{1}$ $\boxed{=}$ $\boxed{/}$ $\boxed{0.001}$ $\boxed{=}$

Problem 2. *Evaluate:*

$$\lim_{x\to 0} \frac{\sin(x) - x}{4x}$$

$$\lim_{x\to 0} \frac{\sin x - x}{4x} = \frac{1}{4} \lim_{x\to 0} \frac{\sin x - x}{x} = \frac{1}{4}\lim_{x\to 0}\left(\frac{\sin x}{x} - \frac{x}{x}\right)$$

$$= \frac{1}{4} \lim_{x\to 0} \left(\underbrace{\frac{\sin x}{x}}_{\to 1} - 1\right) = \frac{1}{4}[1-1] = 0$$



Problem 3. *Let* $f(x)$ *be the function below:*

$$f(x) = \begin{cases} -x+2 & x < -1 \\ mx + b & -1 \le x \le 0 \\ 1 - \sqrt{x} & x > 0 \end{cases}$$

What values do m *and* b *need to be to make* f *continuous for every value of* x*?*

Only Discontinuity that could occur are at $x = -1, 0$

At $x = -1$,
$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-}(-x+2) = 3 \xrightarrow{\text{WANT}} f(-1) = \lim_{x\to -1^+} f(x)$$

Our line needs to contain the point $(-1, 3)$

At $x=0$,
$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(1-\sqrt{x}) = 1 \xrightarrow{\text{WANT}} f(0) = \lim_{x\to 0^-} f(x)$$

Want the point $(0, 1)$

$m = \frac{3-1}{-1-0} = \frac{2}{-1} = -2$

$b = 1$

Def of Continuity

$$\lim_{x \to a} f(x) = f(a)$$

① "Elementary Fcns" are continuous on their domain

② $\frac{p(x)}{q(x)}$ [p, q polynomials] continuous when $q(x) \neq 0$

$$\frac{x+1}{x^2+3x+2}$$

Problem 4. *Use the definition of the derivative to compute $f'(x)$ if $f(x) = x^2 - 1$*

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$[x^2-1]' = \lim_{h\to 0} \frac{(x+h)^2 - 1 - [x^2-1]}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 1 \quad - x^2 + 1}{h} = \lim_{h\to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h\to 0} (2x + h) = 2x$$

$$\boxed{\lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = e^x}$$

Problem 5. *Find the equation of the line tangent to $f(x) = \sqrt{x}$ at $x = 4$*

Power Rule: $[x^p]' = p x^{p-1}$

Slope: $f'(4)$

$$f'(x) = [\sqrt{x}]' = [x^{\frac{1}{2}}]' = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

Point: $(4, f(4)) = (4, \sqrt{4}) = (4, 2)$

Equation: $y - 2 = \frac{1}{4}(x-4)$

$$y = \frac{1}{4}(x-4) + 2$$

$$y = \frac{1}{4}x - 1 + 2$$

$$y = \frac{1}{4}x + 1$$

Problem 6. *Differentiate the following functions:*

(i) $f(x) = 3x^4 + 7x^2 - 5x + 9$

$$f'(x) = 3[x^4]' + 7[x^2]' - 5[x] + [9]'$$
$$= 3(4x^3) + 7(2x) - 5 \cdot 1 + 0$$
$$= 12x^3 + 14x - 5$$

(ii) $g(x) = \underbrace{e^x}_{f}\underbrace{(x^3+1)}_{g}$ Product Rule: $(fg)' = f'g + fg'$

$$f'(x) = [e^x]'(x^3+1) + e^x[x^3+1]' = e^x(x^3+1) + e^x(3x^2)$$
$$= e^x(x^3 + 3x^2 + 1)$$

(iii) $h(x) = \frac{\ln(x)}{\sin(x)}$ Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$\left[\frac{\overbrace{\ln x}^{f}}{\underbrace{\sin x}_{g}}\right]' = \frac{[\ln x]' \sin x - [\ln x][\sin x]'}{[\sin x]^2} = \frac{\frac{\sin x}{x} - \ln x \cos x}{\sin^2 x}$$

(iv) $s(x) = \tan(x^2 + 3x)$ Chain Rule: $[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$$[\tan(x^2+3x)]' = \underbrace{[\tan(x^2+3x)]'}_{\text{w.r.t. } "x^2+3x"} \cdot \underbrace{[x^2+3x]'}_{\text{w.r.t. } x}$$
$$= \sec^2(x^2+3x) \cdot (2x+3)$$

Problem 7. *The position of a dampened pendulum is measured from the angle (in radians) from its resting point. Its position after t seconds is given by the equation:*

$$s(t) = e^{-t}\cos\left(\frac{\pi}{2}t\right)$$

(i) *What is the velocity function $v(t)$ (in radians per second)?*

(ii) *What is the velocity after 1 second?*

a

$$v(t) = s'(t) = [e^{-t}]'\cos\frac{\pi}{2}t + e^{-t}\left[\cos\left(\frac{\pi}{2}t\right)\right]'$$

$$= (-1)e^{-t}\cos\frac{\pi}{2}t + e^{-t}\left[\frac{\pi}{2} \cdot -\sin\frac{\pi}{2}t\right]$$

$$= -e^{-t}\left[\cos\frac{\pi}{2}t + \frac{\pi}{2}\sin\frac{\pi}{2}t\right]$$

b

$$v(1) = -e^{-1}\left(0 + \frac{\pi}{2}\right) = -\frac{\pi}{2e} \quad \text{radians/sec}$$

Problem 8. *Find $\frac{dy}{dx}$ if:*

$$y = (x^2+1)^x$$

$$[\ln(y)]' = [x\ln(x^2+1)]'$$

$$\frac{y'}{y} = \ln(x^2+1) + x \cdot \frac{1}{x^2+1}(2x)$$

$$y' = (x^2+1)^x\left[\ln(x^2+1) + \frac{2x^2}{x^2+1}\right]$$