

MT 2 Review

- Thursday (10/15)
- 8-9 PM
- Lectures 11-18
- WTHR 200 ← Main Floor NOT Balcony

Make Sure to Bring:

- Pencil / Erasers
- PUID
- Calculator TI 30 XA
- Single Line Only

You are expected to know:

- Area / Circumference of a circle
- Surface Area / Volume of a Cube

• We still have class Friday

Balcony Ashton Keith

V1	V2	V3	V4	V5	V6
U1	U2	U3	U4	U5	U7
T1	T2	T3	T4	T5	T7
S1	S2	S3	S4	S5	S7

V7	V8	V9	V10	V11	V12	V13	V14		
U8	U9	U10	U11	U12	U13	U14	U15	U16	U17
T8	T9	T10	T11	T12	T13	T14	T15	T16	T17
S8	S9	S10	S11	S12	S13	S14	S15	S16	S17

V15	V16	V17	V18	V19	V20	
U18	U19	U20	U21	U22	U23	U24
T18	T19	T20	T21	T22	T23	T24
S18	S19	S20	S21	S22	S23	S24

R1	R2	R3	R4	Sarah Anderson		
Q1	Q2	Q3	Q4	Q5	Q6	Q7
P1	P2	P3	P4	P5	P6	P7
O1	O2	O3	O4	O5	O6	O7
N1	N2	N3	N4	N5	N6	N7
M1	M2	M3	M4	M5	M6	M7
L1	L2	L3	L4	L5	L6	L7
K1	K2	K3	K4	K5	K6	K7
J1	J2	J3	J4	J5	J6	J7
I1	I2	I3	I4	I5	I6	I7
H1	H2	H3	H4	H5	H6	H7

G1	G2	G3	G4	G5	G6	G7
F1	F2	F3	F4	F5	F6	F7
E1	E2	E3	E4	E5	E6	E7
D1	D2	D3	D4	D5	D6	D7
C1	C2	C3	C4	C5	C6	C7
B1	B2	B3	B4	B5	B6	B7
A1	A2	A3	A4	A5	A6	A7

Main Floor

Liam Ashton

P8	X	X	X	X	P9				
O8	O9	O10	O11	O12	O13	O14	O15	O16	O17
N8	N9	N10	N11	N12	N13	N14	N15	N16	N17
M8	M9	M10	M11	M12	M13	M14	M15	M16	M17
L8	L9	L10	L11	L12	L13	L14	L15	L16	L17
K8	K9	K10	K11	K12	K13	K14	K15	K16	K17
J8	J9	J10	J11	J12	J13	J14	J15	J16	J17
I8	I9	I10	I11	I12	I13	I14	I15	I16	I17
H8	H9	H10	H11	H12	H13	H14	H15	H16	H17
G8	G9	G10	G11	G12	G13	G14	G15	G16	G17
F8	F9	F10	F11	F12	F13	F14	F15	F16	F17
E8	E9	E10	E11	E12	E13	E14	E15	E16	E17
D8	D9	D10	D11	D12	D13	D14	D15	D16	D17
C8	C9	C10	C11	C12	C13	C14	C15	C16	C17
B8	B9	B10	B11	B12	B13	B14	B15	B16	B17
X	A8	A9	A10	A11	A12	A13	X		

				R5	R6	R7	R8
Q8	Q9	Q10	Q11	Q12	Q13	Q14	
P10	P11	P12	P13	P14	P15	P16	
O18	O19	O20	O21	O22	O23	O24	
N18	N19	N20	N21	N22	N23	N24	
M18	M19	M20	M21	M22	M23	M24	
L18	L19	L20	L21	L22	L23	L24	
K18	K19	K20	K21	K22	K23	K24	
J18	J19	J20	J21	J22	J23	J24	
I18	I19	I20	I21	I22	I23	I24	
H18	H19	H20	H21	H22	H23	H24	
G18	G19	G20	G21	G22	G23	G24	
F18	F19	F20	F21	F22	F23	F24	
E18	E19	E20	E21	E22	E23	E24	
D18	D19	D20	D21	D22	D23	D24	
C18	C19	C20	C21	C22	C23	C24	
B18	B19	B20	B21	B22	B23	B24	
A14	A15	A16	A17	A18	A19	A20	

1:30 Section

WTHR 200
480 stations (41 LH)
Exam Capacity 260 stations (shown shaded)

2:30 Section

SEE IMPORTANT MARKING INSTRUCTIONS ON SIDE 2

LAST NAME	FIRST NAME	MI
PENCE	ZACHARA	

T	F
1	A B C D E F
T	F
2	A B C D E F
T	F
3	A B C D E F
T	F
4	A B C D E F
T	F
5	A B C D E F
T	F
6	A B C D E F
T	F
7	A B C D E F
T	F
8	A B C D E F
T	F
9	A B C D E F
T	F
10	A B C D E F
T	F
11	A B C D E F
T	F
12	A B C D E F
T	F
13	A B C D E F
T	F
14	A B C D E F
T	F
15	A B C D E F

COMPLETE ALL INFORMATION AS DIRECTED

SECTION NUMBER	TEST/QUIZ NUMBER	STUDENT IDENTIFICATION NUMBER
027	1234	00##

SIGNATURE 30P Sign your name

DATE	COURSE	INSTRUCTOR

Should start with two zeros

This is an optional assignment that will be worth 2 points of extra credit. You must show work to get credit.

NOTE: These are just review problems of Lectures 11-18; these are not necessarily representative of the problems of the exam. The exam can (and most likely will) have different problems.

Directions:

1. **For credit, you only need to complete 2 problems** on the next page; though, I encourage you to at least attempt every problem. Clearly mark the question number and your final answer.
2. **Submitting the Assignment:** Scan your work and save it as a PDF file (any other filetype is **not** allowed). Submit the assignment electronically using the Qualtrics Survey Link (the link is on the assignment on Brightspace).
3. The answers will be given in the lecture after the due date (10/14). Therefore, **no late submissions will be allowed.**

Topics of Exam 2:

- **Lecture 11:** The Chain Rule; Derivative of the Natural Logarithmic Function
- **Lecture 12:** Higher Order Derivatives
- **Lecture 13:** Implicit Differentiation
- **Lecture 14-15:** Related Rates
- **Lecture 16:** Relative Extrema and Critical Numbers
- **Lecture 17:** Increasing and Decreasing Functions and the First Derivative Test
- **Lecture 18:** Concavity; Inflection Points and the Second Derivative Test

Problem 1. Find $f'(1)$ if:

$$f(x) = \ln \left[\frac{4+4x}{4-x} \right]^{\frac{1}{4}} = \frac{1}{4} \ln \left[\frac{4+4x}{4-x} \right]$$
$$= \frac{1}{4} \left[\ln(4+4x) - \ln(4-x) \right]$$
$$= \frac{1}{4} \ln(4+4x) - \frac{1}{4} \ln(4-x)$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \text{and} \quad \frac{d}{dx}[\ln(f(x))] = \frac{f'(x)}{f(x)}$$

$$f'(x) = \frac{1}{4} \cdot \frac{1}{4+4x} \cdot 4 - \frac{1}{4} \cdot \frac{1}{4-x} \cdot (-1)$$

$$f'(x) = \frac{1}{4+4x} + \frac{1}{4(4-x)}$$

$$f'(1) = \frac{1}{8} + \frac{1}{12} = \frac{3}{24} + \frac{2}{24} = \frac{5}{24}$$

Problem 2. Find the initial acceleration of a particle if its position after t seconds is given by:

$$s(t) = \ln(t^2 + 1) = \ln(t^2 + 1)$$

Express your final answer in $\frac{\text{m}}{\text{s}^2}$.

$$a(t) = v'(t) = [s'(t)]' = s''(t)$$

I.e., we need to find $a(0) = s''(0)$

$$v(t) = s'(t) = \frac{1}{t^2 + 1} \cdot 2t = \frac{2t}{t^2 + 1}$$

$$a(t) = v'(t) = \frac{2(t^2 + 1) - 2t(2t)}{(t^2 + 1)^2} \quad \frac{\text{m/s}}{\text{s}}$$

$$a(0) = \frac{2(0 + 1) - 0}{(0 + 1)^2} = \frac{2}{1} = 2 \text{ m/s}^2$$

Problem 3. Given the algebraic curve $x^2 + 4xy + y^2 = 13$. What is the equation of the line tangent to the curve at the point $(x, y) = (2, 1)$?

$$x^2 + 4xy + y^2 = 13$$

$4xf(x) \quad [f(x)]^2$

Find $\frac{dy}{dx}|_{(2,1)}$; $\frac{d}{dx} [x^2 + 4xy + y^2] = \frac{d}{dx} [13]$

$$2x + 4y + 4x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$4x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - 4y$$

$$[4x + 2y] \frac{dy}{dx} = -2x - 4y$$

$$\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y}$$

$$\frac{dy}{dx}|_{(2,1)} = \frac{-2(2) - 4(1)}{4(2) + 2(1)}$$

$$= \frac{-8}{10} = \underline{\underline{-\frac{4}{5}}}$$

Point: (2,1)

Eg:

$$y - 1 = -\frac{4}{5}(x - 2)$$

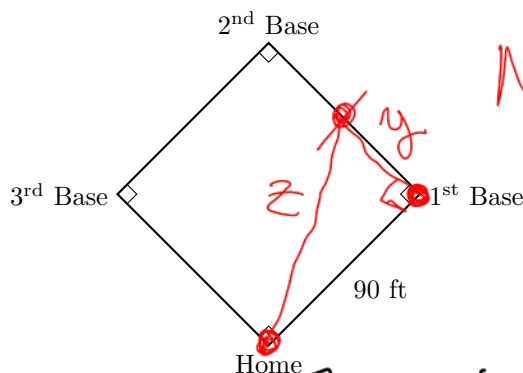
$$y - 1 = -\frac{4}{5}x + \frac{8}{5}$$

$$y = -\frac{4}{5}x + \frac{13}{5}$$

Problem 4. A baseball diamond is a square with 90 ft on each side. A player runs from first base to second base at 13 ft/sec. At what rate is the player's distance from home base increasing when he is half way from first to second base?

Formula:
 $90^2 + y^2 = z^2$

K: $\frac{dy}{dt} = 13$



NTK:

$\frac{dz}{dt}$ when $y = 45$

$$\frac{d}{dt} [90^2 + y^2] = \frac{d}{dt} [z^2]$$

$$2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{2y}{2z} \frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{y}{z} \frac{dy}{dt}$$

Find z when $y = 45$:

$$90^2 + y^2 = z^2$$

$$90^2 + 45^2 = z^2$$

$$90^2 + (\frac{1}{2} \cdot 90)^2 = z^2$$

$$1 \cdot 90^2 + \frac{1}{4} \cdot 90^2 = z^2$$

$$z^2 = \frac{5}{4} \cdot 90^2$$

$$z = \frac{\sqrt{5}}{2} \cdot 90 = 45\sqrt{5}$$

$$\frac{dz}{dt} = \frac{y}{z} \frac{dy}{dt} = \frac{45}{45\sqrt{5}} (13) = \frac{13}{\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}} \right) = \frac{13}{5} \sqrt{5} \frac{\text{ft}}{\text{s}}$$

Problem 5. (OPTIONAL...very optional) Repeat Problem 4, but replace 13ft/s with an arbitrary speed v ft/s and 90ft with an arbitrary length ℓ ft.

$$\frac{22}{15}$$

Problem 6. Sand pours onto a surface at $19 \text{ cm}^3/\text{s}$, forming a conical pile with a base diameter that is always equal to the pile's altitude. How fast is the altitude of the pile increasing when the pile is 5 cm high?

[NOTE: The volume of a cone with base radius r and height h is $V = \frac{1}{3}\pi r^2 h$]

Formula:

$$V = \frac{1}{3}\pi r^2 h$$



NTK: $\frac{dh}{dt}$ when $h = 5$

K: $\frac{dV}{dt} = 19$

$$2r = h$$

$$r = \frac{h}{2}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{\pi}{12} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{\pi h^2} \frac{dV}{dt}$$

$\uparrow$ 5 $\uparrow$ 19

$$= \frac{76}{25\pi} \frac{\text{cm}}{\text{s}}$$

Problem 7. Find the critical numbers of $y = x^2e^x$.

Problem 8. For the function $f(x) = x^3 - 6x^2 - 15x + 4$:

- When is f increasing and concave upward?
- When is f increasing and concave downward?
- When is f decreasing and concave upward?
- When is f decreasing and concave downward?
- State the locations (x -coordinates) of any relative maximums, relative minimums, and inflection points.

Make a sign chart for f' and f''

$$\begin{aligned} f'(x) &= 3x^2 - 12x - 15 \\ &= 3(x^2 - 4x - 5) \\ &= 3(x+1)(x-5) \stackrel{\text{set}}{=} 0 \end{aligned}$$



$\Rightarrow x = -1, 5$ are our critical numbers

$$\begin{aligned} f''(x) &= 6x - 12 \\ &= 6(x-2) \stackrel{\text{set}}{=} 0 \end{aligned}$$



$\Rightarrow x = 2$ is a candidate for the loc. of an inflection point

Test Point	-100	-1	0	2	(3)	5	100
Sign of $(x+1)$	-		+		(+)		+
Sign of $(x-5)$	-		-		(-)		+
Sign of 3	+		+		(+)		+
Sign of f'	+		-		-		+

Sign of $6(x-2)=f''$	$-$	$-$	$+$	$+$
Result of I/D Test	Inc	Dec	Dec	Inc
Result of C Test	CD	CD	CU	CU
	$-1 \in \text{rel max}$		$2 \in \text{IP}$	$5 \in \text{rel min}$

Conclusion:

- On $(-\infty, -1)$, f is inc and CD
- On $(-1, 2)$, f is dec and CD
- On $(2, 5)$, f is dec and CU
- On $(5, \infty)$, f is inc and CU

Verify the rel. ext. using the 2nd $\frac{d}{dx}$ test

$$f''(x) = 6(x-2)$$

$$f''(-1) = -18 < 0 \quad \text{rel max}$$

$$f''(5) = 18 > 0 \quad \text{rel min}$$