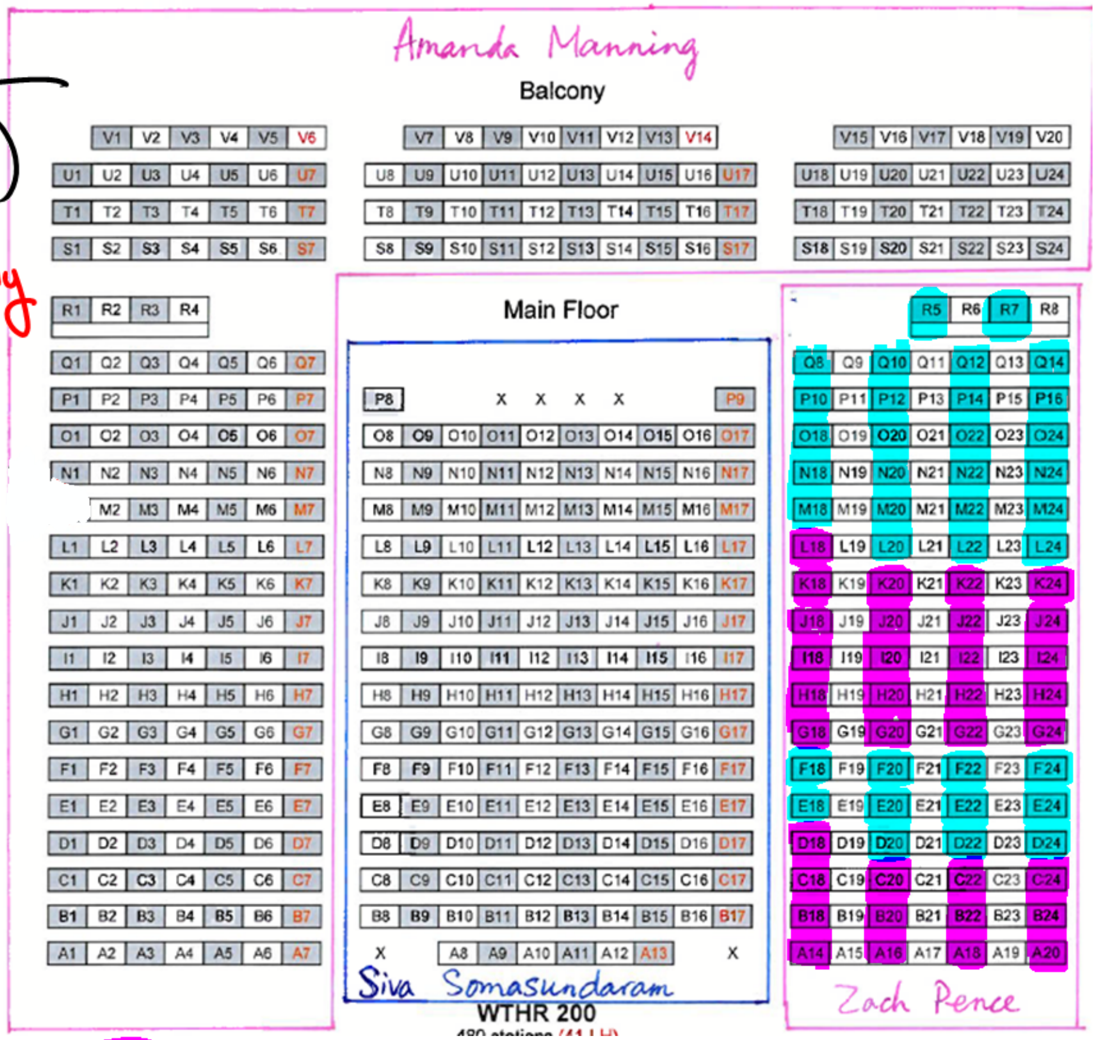
MT 2 Review

- Tomorrow (Tues. 10/21)
- 8-9 PM
- WTHR 200 ← Main Floor <u>NOT</u> Balcony
- Lectures 11-19

Make sure to bring

- Pencils / Erasers
- PUID
- Calculator TI 30XA <u>Single line only</u>



7:30 Section   8:30 Section

Exam 2 Review

① Find crit. nums. of $y = [\ln(x+4)]^2$

Crit. Nums. : when $y' = 0$ or undefined

$$y' = 2\ln(x+4) \cdot \frac{1}{x+4} \cdot (1) = \frac{2\ln(x+4)}{x+4} \overset{\text{set}}{=} 0$$

$$2\ln(x+4) = 0$$

$$\ln(x+4) = 0$$

$$e \qquad e$$

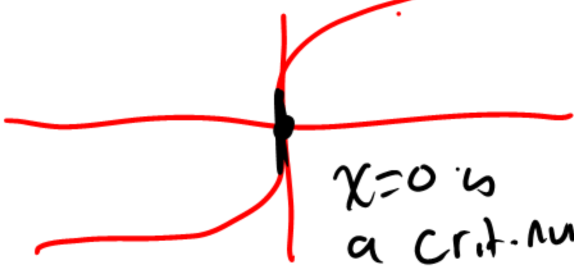
$$x+4=1$$

$$\boxed{x=-3}$$ ← Only crit. num.

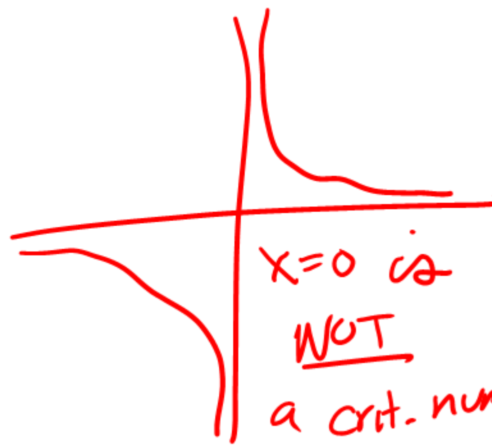
$x=-4$? No

Remark Critical Numbers need to be in the domain

$\sqrt[3]{x}$



$x=0$ is a crit. num



$x=0$ is NOT a crit. num.

② Find $\left.\frac{dy}{dx}\right]_{(x,y)=(2,1)}$ given

$$\frac{d}{dx}\left(x^2+4xy_{(x)}+[y]^2-13\right)=\frac{d}{dx}(0)$$

$$2x+\frac{d}{dx}(4x)\,y+4x\frac{d}{dx}(y)+2y\frac{dy}{dx}-0=0$$

$$\underbrace{2x+4y}+4x\frac{dy}{dx}+2y\frac{dy}{dx}=0$$

$$4x\frac{dy}{dx}+2y\frac{dy}{dx}=-(2x+4y)$$

$$\frac{d}{dx}\underbrace{(4x + 2y)} = -(2x + 4y)$$

$$\frac{dy}{dx} = -\frac{2x + 4y}{4x + 2y}$$

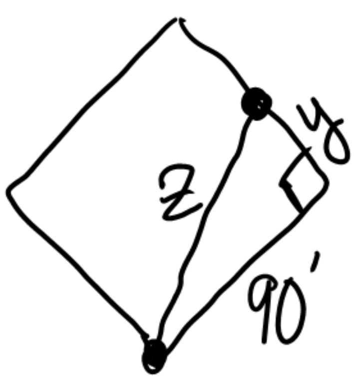
$$\left.\frac{dy}{dx}\right|_{(2,1)} = -\frac{2(2) + 4(1)}{4(2) + 2(1)}$$

$$= -8/10 = \boxed{-4/5}$$

Recall $\ln(y) = \ln(y_{(x)})$

$$\frac{d}{dx}(\ln y) = \frac{1}{y} \cdot \frac{dy}{dx}$$

③



Know: $\frac{dy}{dt} = 13$. Length $= 90$ ft

Need to know: $\frac{dz}{dt}$ when $y = 45$

$$\frac{d}{dt}\left(90^2 + [y_{(t)}]^2\right) = \frac{d}{dt}\left([z_{(t)}]^2\right)$$

$$2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{\partial y}{\partial z}\frac{dy}{dt} = \frac{y}{z}\frac{dy}{dt}$$

Find z when $y = 45$: $90^2 + y^2 = z^2$

$z = \sqrt{90^2 + y^2}$. Hence, $z\Big]_{y=45} = \sqrt{90^2 + 45^2}$

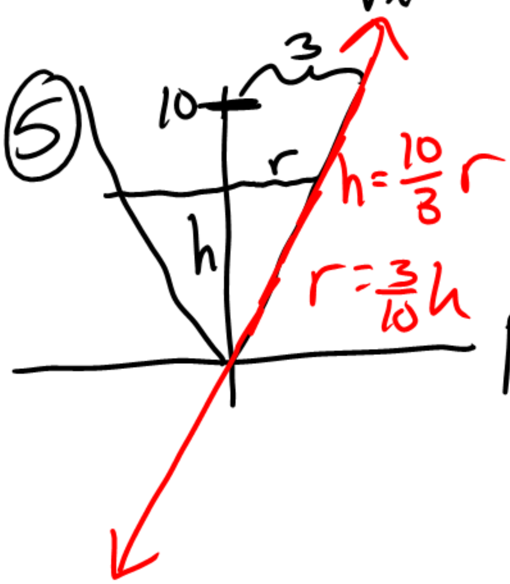
$$= \sqrt{(45\cdot 2)^2 + (45\cdot 1)^2} = \sqrt{45^2(2^2 + 1^2)} = 45\sqrt{5}$$

$$\frac{dz}{dt}\Big]_{\substack{y=45\\ y'=13}} = \frac{45}{45\sqrt{5}}(13) = \boxed{\frac{13}{\sqrt{5}}\ \frac{ft}{s}}$$

4) $$\frac{dz}{dt} = \frac{y}{z}\frac{dy}{dt} = \frac{y}{\sqrt{\ell^2 + y^2}}\frac{dy}{dt}$$

$$\frac{dz}{dt}\Big]_{\substack{y=\frac{1}{2}\ell\\ y'=v}} = \frac{\frac{1}{2}\ell}{\sqrt{\ell^2 + \frac{1}{4}\ell^2}}\, v = \dots = \frac{v}{\sqrt{5}}\ ft/s$$

5)



Know: $\frac{dV}{dt} = 2\ cm^3/s$

NTK: $\frac{dh}{dt}$ when $h = 5$

$$V = \frac{\pi}{3} r^2 h$$

$$V = \frac{\pi}{3}\left(\frac{3}{10}h\right)^2 h = \frac{3\pi}{100}h^3$$

$$\frac{dV}{dt} = \frac{9\pi}{100}h^2\,\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{100}{9\pi h^2}\,\frac{dV}{dt}$$

$$\left.\frac{dh}{dt}\right]_{\substack{h=5\\ V'=2}} = \frac{100}{9\pi(5)^2}\,2 = \boxed{\frac{8}{9\pi}\ \text{cm/s}}$$

⑥ $f(x) = x^3 - 6x^2 - 15x + 4$

Step1 Find candidates for rel. max/mins/ IPs

$$f'(x) = 3x^2 - 12x - 15 \overset{\text{set}}{=} 0$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0 \Rightarrow x = -1, 5$$

$$f''(x) = 6x - 12$$

$$6(x-2) \overset{\text{set}}{=} 0 \Rightarrow x = 2$$

Step 2 Make Sign Chart

$-1 \qquad 2 \qquad 5$

Test Points	-10	0	3	10
Sign of f'	$+$	$-$	$-$	$+$
Sign of f''	$-$	$-$	$+$	$+$
Result of I/D Test	Inc	Dec	Dec	Inc
Result of C Test	CD	CD	CU	CU

-1 Rel. Max

2 I.P.

5 Rel. Min

(7) Find abs. max / min given $f(x) = 10 + 27x - x^3$ on $[0,4]$

$$f'(x) = 27 - 3x^2 \overset{\text{set}}{=} 0$$

$$9 - x^2 = 0$$

$$(3+x)(3-x) = 0$$

$$\Rightarrow \quad x = \pm 3 \xrightarrow{-3 \text{ is not in } [0,4]} x = 3$$

x	3	0	4
$f(x)$	64	10	54

$x=0$ is abs. min

$x=3$ is abs. max

Unnecessary here, but use the 2^{nd} $\frac{d}{dx}$ test to verify $x=3$ is a rel. max.

$$f'(x) = 27-3x^2$$

$$f''(x) = -6x$$

$$f''(3) = -6(3) = -18 < 0$$

$f''<0 \Rightarrow f$ is CD near $3 \Rightarrow$ $x=3$ is the loc. of a rel. max