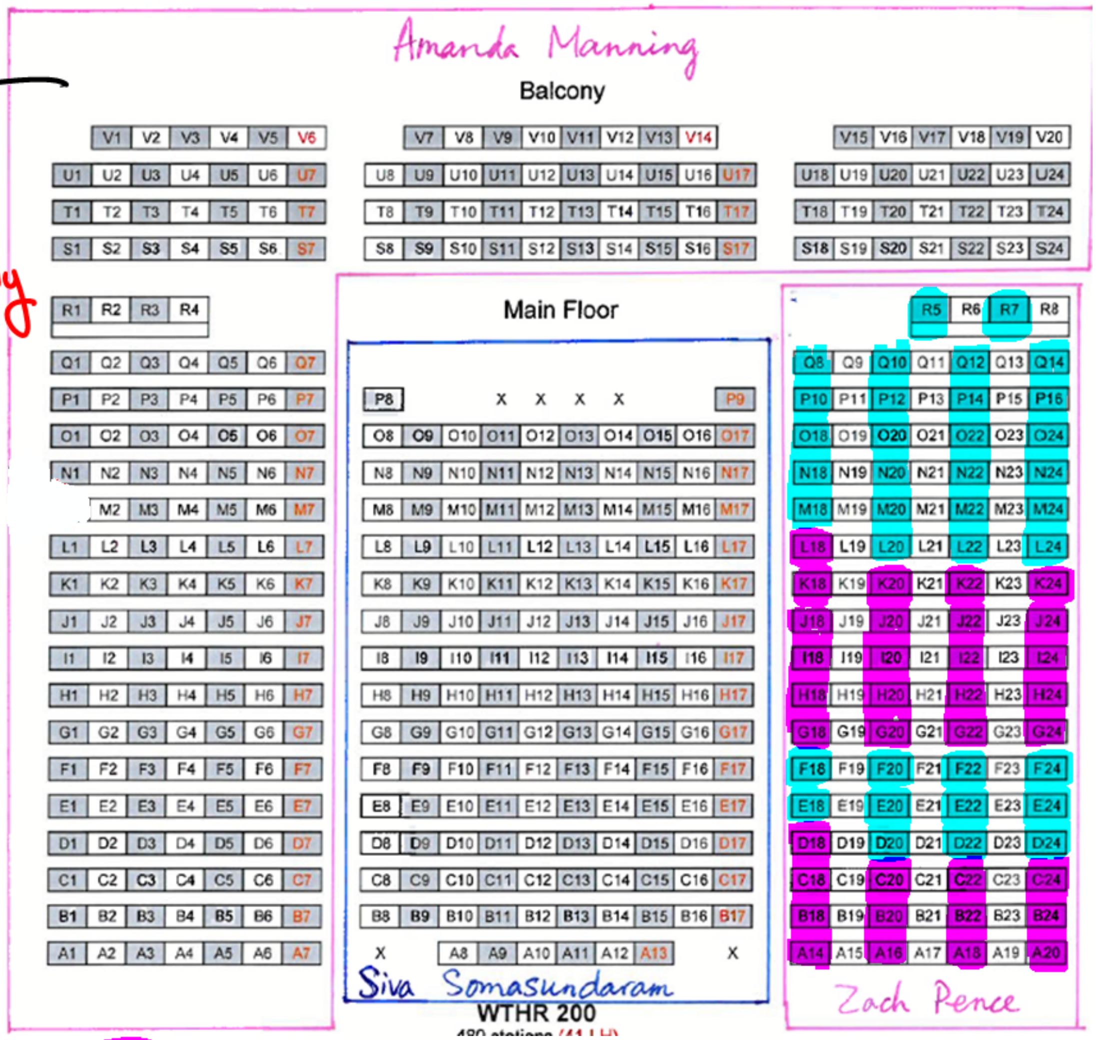
MT 3 Review

- Monday (11/10)
- 8-9 PM
- WTHR 200 — Main Floor NOT Balcony
- Lectures 20-28

Make sure to bring

- Pencils / Erasers
- PUID
- Calculator TI 30XA Single line only



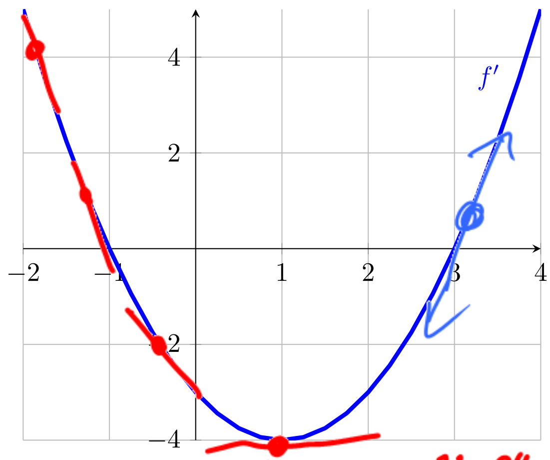
7:30 Section 8:30 Section

Problem 1. *A graph of f' is given below.*

(a) *Determine when f is increasing and when it is decreasing.*

(b) *Determine when f is concave up and when it is concave down.*

(c) *Locate the positions (x-coordinates) of any relative extrema and inflection points.*



↑ where $[f']' = f'' = 0$

@ Inc / Dec

$f'>0$ $f'<0$ $f'>0$

-1 3

Inc: $(-\infty, -1) \cup (3, \infty)$

Dec: $(-1, 3)$

Rel Max @ $x=-1$

Rel Min @ $x=3$

Concavity:

$f''<0 \Rightarrow CD$ $f''>0 \Rightarrow CU$

1

CD on $(-\infty, 1)$

CU on $(1, \infty)$

Inflection Point at $x=1$

Problem 2. *Compute* $\lim_{x\to\infty} \frac{x^2+1}{2-x}$ *and* $\lim_{x\to-\infty} \frac{x^2+1}{2-x}$.

$$\frac{x^2+1}{-x+2} = \frac{x\left(x+\frac{1}{x}\right)}{x\left(-1+\frac{2}{x}\right)} = \frac{x+\frac{1}{x}}{-1+\frac{2}{x}}$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x+\frac{1}{x}}{-1+\frac{2}{x}} = -\infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x+\frac{1}{x}}{-1+\frac{2}{x}} = \infty$$

Problem 3. *Use the techniques learned in Lecture 22 to sketch a graph of the function* $y = \frac{x^2}{x+8}$.

$\frac{x^2}{x+8}$ ← deg 2 ; ↑ deg 1

Domain: $\mathbb{R} \setminus \{-8\}$

Intercepts: $(0,0)$. Solve $\frac{x^2}{x+8} = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$

Asymptote: VA @ $x = -8$

$$\begin{array}{r} x - 8 \\ x+8 \overline{) x^2 } \\ -(x^2+8x) \\ \hline -8x \\ -(-8x-64) \\ \hline 64 \end{array}$$

Slant Asymptote of $y = x - 8$

Rel. Max / Mins

$$f(x) = \frac{x^2}{x+8}$$

$$f'(x) = \frac{2x(x+8) - x^2}{(x+8)^2} = \frac{x^2 + 16x}{(x+8)^2}$$

$$\overset{\text{set}}{=} 0$$

$$\Rightarrow x^2 + 16x = 0$$

$$x(x+16) = 0 \Rightarrow \boxed{x = -16, 0}$$

$$f'(x) = \frac{x^2 + 16x}{(x+8)^2}$$

$$f''(x) = \frac{(2x+16)(x+8)^2 - (x^2+16x)2(x+8)}{(x+8)^4}$$

$$f''(x) = \frac{72(x+8)}{(x+8)^4}$$

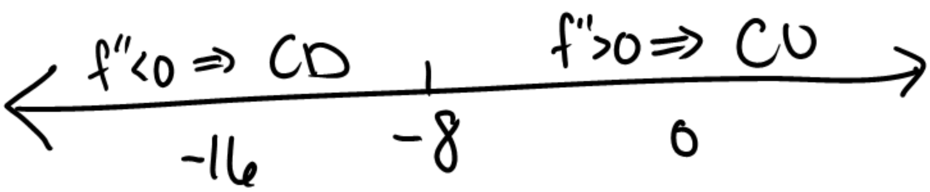
Determine Max/Min by 2^{nd} $\frac{d}{dx}$ test:

$f''(-16) < 0 \Rightarrow$ Local Max at $x = -16$

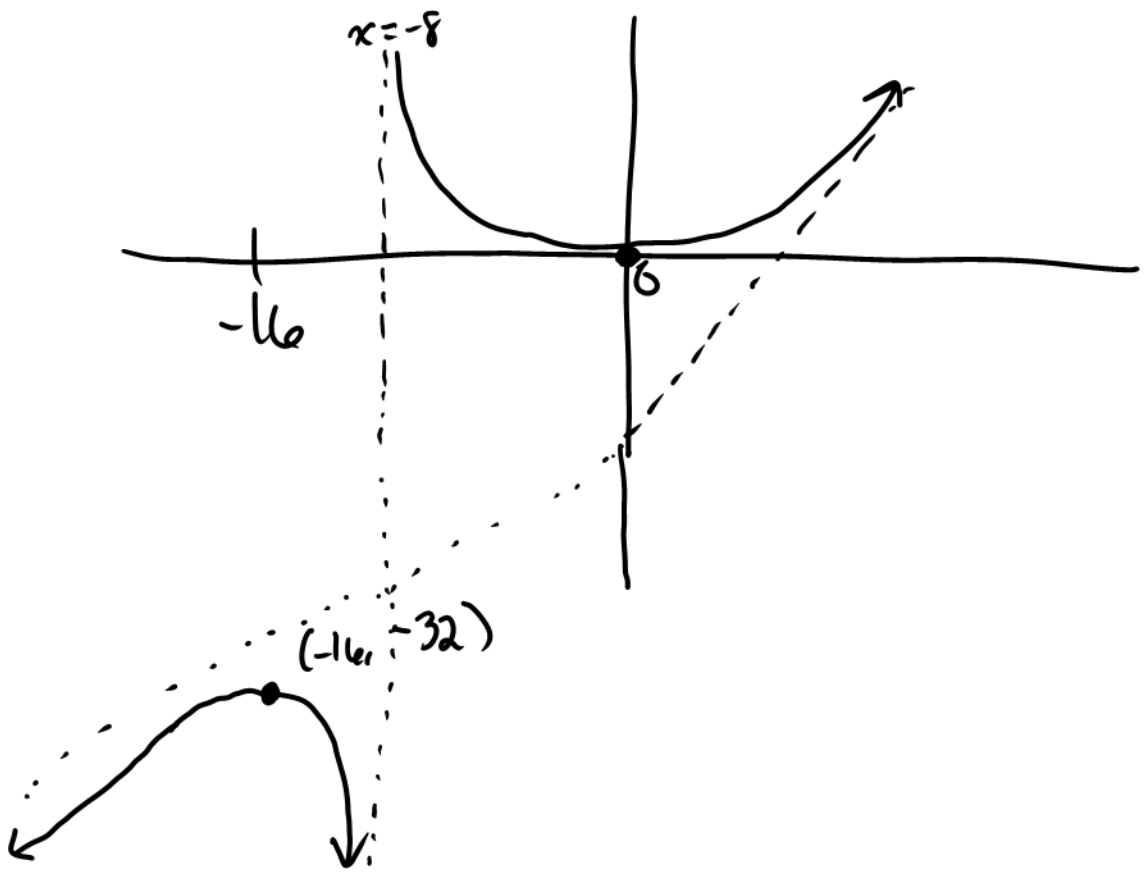
$f''(0) > 0 \Rightarrow$ Local Min at $x = 0$

Concavity / IPs:

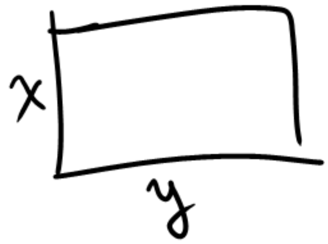
$$f''(x) \overset{set}{=} 0 \iff 72(x+8) = 0 \Rightarrow x = -8$$



Not IP, but concavity can change



Problem 4. *If a rectangle has a fixed perimeter of 40, what is its maximum area?*



Obj: Max $A_{(x,y)} = xy$

Given: $40 = 2x + 2y$

$20 = x + y$

$y = 20 - x$

$$A_{(x)} = x(20-x) = 20x - x^2$$

$$A'(x) = 20 - 2x \overset{set}{=} 0$$

$$x = 10$$

Verify this is the location of an abs. max:

$$A''(x) = -2$$

$$A''(10) = -2 < 0 \Rightarrow \text{Local Max @ } x = 10$$

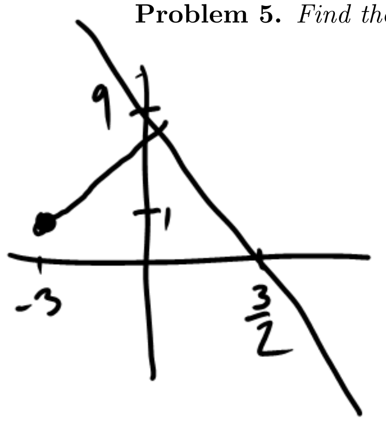
Local Max + Only 1 Crit. Num. $\Rightarrow$ Abs. Max

Maximum Area?

$$A(10) = (10)(20-10) = 10^2 = 100$$

$d = \sqrt{(x-x_0)^2 + (y-y_0)^2}$

Problem 5. *Find the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$.*



Obj: Min $S(x,y) = d^2(x,y) = (x+3)^2 + (y-1)^2$

Given: $6x + y = 9$

$y = -6x + 9$

$$S(x) = (x+3)^2 + (-6x+9-1)^2 = (x+3)^2 + (-6x+8)^2$$

$$= 37x^2 - 90x + 73$$

$$S'(x) = 74x - 90 \overset{\text{set}}{=} 0$$

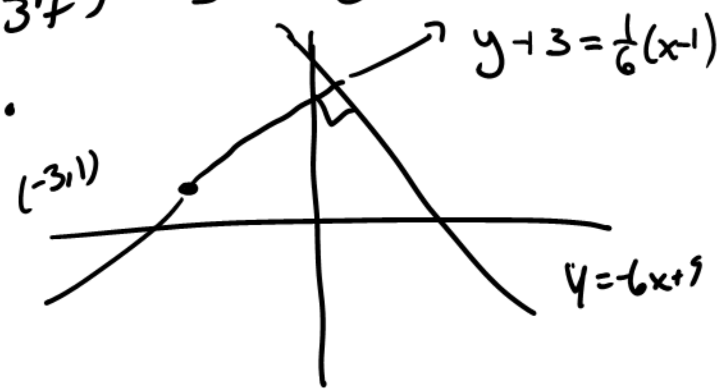
$$x = \frac{90}{74} = \frac{45}{37}$$

$x = \frac{45}{37}$ is the loc. of a rel. min

It is indeed the location of a min.

$$y = -6\left(\frac{45}{37}\right) + 9 = \frac{63}{37}$$

Conclusion: The point $\left(\frac{45}{37}, \frac{63}{37}\right)$ is closest to the point $(-3,1)$.



Problem 6. *A particle is moving through space; its acceleration function is given by* $a(t) = \cos t + \sin t$ *for* $t \geq 0$. *Find the position of the particle at time* t *when* $s(0) = 0$ *and* $v(0) = 5$.

Need to solve the IVP

$$\begin{cases} a(t) = \cos t + \sin t & \leftarrow s'' \\ v(0) = 5 & \leftarrow s' \\ s(0) = 0 \end{cases}$$

$$a(t) = \cos t + \sin t$$

$$\int a(t)\,dt = \int (\cos t + \sin t)\,dt$$

$$v(t) = \sin t - \cos t + C$$

$v(0) = 5$

$$5 = v(0) = \sin 0 - \cos 0 + C$$

$$5 = -1 + C$$

$$C = 6$$

$$v(t) = \sin t - \cos t + 6$$

$$\int v(t)\,dt = \int (\sin t - \cos t + 6)\,dt$$

$$S(t) = -\cos t - \sin t + 6t + D$$

$s(0) = 0$:

$$0 = s(0) = -1 - 0 + 0 + D$$

$$0 = -1 + D$$

$$D = 1$$

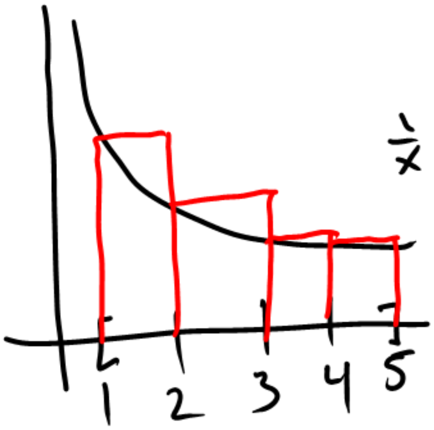
$$\boxed{S(t) = -\cos t - \sin t + 6t + 1}$$

Problem 7. *We estimate the area underneath the graph of* $f(x) = \frac{1}{x}$ *on the interval* $[1, 5]$.

(a) *Compute the left Riemann sum with 4 rectangles. Is this an overestimate or an underestimate? Explain why (The exact area is* $\ln 5$*, but you don't need to know that to answer the question).*

(b) *Repeat (a) using the right Riemann sum.*

(c) *Set up (but do not compute) the left Riemann sum using N rectangles (where N is a positive integer).*



ⓐ $\Delta x = \frac{5-1}{4} = \frac{4}{4} = 1$

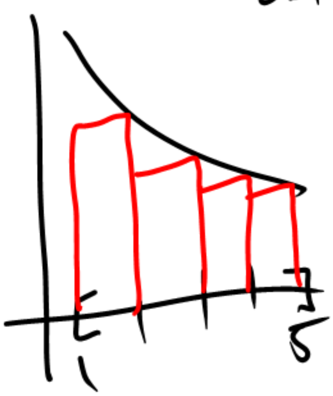
$$L_4 = \sum_{i=0}^{3} f(a + i\Delta x)\,\Delta x$$

$$= \sum_{i=0}^{3} \frac{1}{1+i}(1) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

$$= \frac{25}{12} \leftarrow \text{Over estimate}$$

$\Delta x = 1$

ⓑ $R_4 = \sum_{i=1}^{4} \frac{1}{1+i} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60} \Leftarrow$ Under estimate



ⓒ $\Delta x = \frac{5-1}{N} = \frac{4}{N}$

$$L_N = \sum_{i=0}^{N-1} \frac{1}{1+\left(\frac{4}{N}\right)i}\left(\frac{4}{N}\right)$$