MA 161 ONLINE, SUMMER 2026
MIDTERM EXAM 1

Name (Print): Key ____________________ PUID: ____________________

Section Number: ____________________ Name of TA: ____________________

Start Time: ____________________ End Time: ____________________

Directions:

- Please use a **pen** if possible. Marks made by pen scan better than marks made by pencil.
- This is a **one-hour** exam.
- There are **12 questions** worth 8 points each; the maximum score is 100 points.
- The remaining 4 points are earned for filling out the information at the top of the page correctly. Section numbers, and their associated TA, are provided in the table below.

203	June Marie Weiland	**204**	Laura Maria Castillo

- Circle one and only one choice for each multiple-choice problem (Problems 1-3). **No partial credit** will be given on multiple-choice problems.
- Show all relevant work on each free-response problem (Problems 4-12). You must include some work or justification to receive full credit on these problems; partial credit may be given for steps leading to the correct solution. **Write the final answer in the box(es) provided**.
- Any external help or communication is not allowed, except with the proctor for logistical reasons. This exam is closed book and closed notes. **Calculators are not allowed**. Electronic devices, including smart watches, are to be turned off and put away.
- The last sheet is scratch paper. You may use only the scratch paper provided in this exam booklet. No additional scratch paper is permitted.

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

I agree to abide by the instructions above:

Student Signature: ____________________

Proctor Signature: ____________________

Question	Points	Score
1	8	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	8	
12	8	
Correct Header Info	4	
Total:	100	

1. [8 points] Simplify $\cos(\sin^{-1} x)$.

   A. x

   B. $\sqrt{1-x^2}$

   C. $\frac{x}{\sqrt{1-x^2}}$

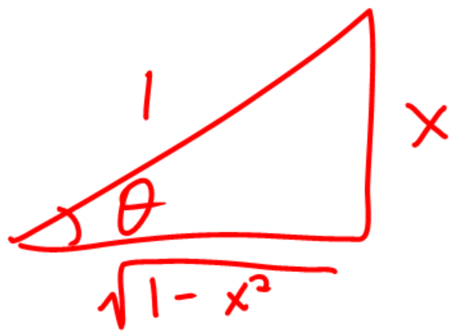
   D. $\frac{1}{\sqrt{1-x^2}}$

   E. $\frac{\sqrt{x^2-1}}{x}$

Let $\theta = \sin^{-1} x$

$\frac{x}{1} = \sin\theta$

Make a Triangle



So,

$$\cos(\sin^{-1} x) = \cos\theta = \frac{\sqrt{1-x^2}}{1} = \boxed{\sqrt{1-x^2}}$$

2. [8 points] When is $f(x)$ discontinuous?

$$f(x) = \begin{cases} -1 & x < -1 \\ \frac{x^2+x}{2x} & -1 \leq x \leq 1 \\ -\cos \pi x & x > 1 \end{cases}$$

A. $x = 0$

B. $x = -1$

C. $x = -1$ and $x = 1$

D. $x = 0$ and $x = -1$

E. f is continuous everywhere.

It is continuous at 1:

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} [-\cos \pi x] = -\cos \pi = -(-1) = 1$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} \frac{x^2+x}{2x} = 1$$

$$f(1) = \frac{1^2+1}{2(1)} = 1$$

There is a removable discontinuity at $x=0$:

$$f(x) = \frac{x^2+x}{2x} = \frac{x(x+1)}{2x}$$

Not defined at 0, but $\lim_{x\to 0} f(x) = \frac{1}{2}$ exists

There is a jump discontinuity at $x=-1$:

$$\lim_{x\to -1^-} f(x) = -1$$

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} \frac{x^2+x}{2x} = \frac{(-1)^2+(-1)}{2(-1)} = 0$$

3. [8 points] Evaluate the limit:

$$\lim_{x\to 0} x^5 \cos\left(\frac{2}{x}\right)$$

A. $-\infty$

B. ∞

C. -1

D. 1

E. 0

Since $-1 \leq \cos x \leq 1$

$$-x^5 \leq x^5 \cos\left(\tfrac{2}{x}\right) \leq x^5$$

$-x^5 \xrightarrow{x\to 0} 0$, $\quad x^5 \cos\left(\tfrac{2}{x}\right) \to 0$, $\quad x^5 \xrightarrow{x\to 0} 0$

by Squeeze theorem

4. [8 points] When a camera's flash goes off, the batteries immediately begin to recharge the capacitor inside the camera. The amount of charge stored t seconds after the flash is given by:

$$Q(t) = Q_0(1 - e^{-\frac{t}{2}})$$

where Q_0 is the maximum capacity. How long after the flash does it take for the capacitor to recharge to 90% capacity.

$$Q_0(1-e^{-\frac{t}{2}}) \overset{\text{set}}{=} 0.9\, Q_0$$

$$1-e^{-\frac{t}{2}} = 0.9$$

$$e^{-\frac{t}{2}} = 0.1$$

$$\ln(e^{-\frac{t}{2}}) = \ln(0.1)$$

$$-\frac{t}{2} = \ln(0.1)$$

$$t = -2\ln(0.1)$$

Optionally,

$$t = -2\ln\left(\frac{1}{10}\right)$$

$t = 2\ln(10)$ seconds

$t =$ $2 \ln 10$ seconds

5. [8 points] Let ℓ be the line tangent to the curve $g(x) = (3x - x^2)(3 - x - x^2)$ at $x = 1$. Where does ℓ intersect the $x-$axis?

$$g'(x) = [3x - x^2]'(3 - x - x^2) + (3x - x^2)[3 - x - x^2]'$$

$$g'(x) = [3 - 2x](3 - x - x^2) + (3x - x^2)[-1 - 2x]$$

$$g'(1) = [3-2](3-1-1) + (3-1)[-1-2]$$
$$= -5$$

We also need $g(1)$:

$$g(1) = (3-1)(3-1-1) = 2$$

So, ℓ is the line

$$y - g(1) = g'(1)(x-1)$$
$$y - 2 = -5(x-1)$$

Find x-intercept of ℓ:

$$-2 = -5(x-1)$$
$$\frac{2}{5} = x - 1$$
$$x = \frac{7}{5}$$

Location of Intersection: $(\frac{7}{5}, 0)$

6. [8 points] Evaluate the limit:

$$\lim_{h\to 0} \frac{\sqrt[3]{\frac{1}{64+h}} - \frac{1}{4}}{h}$$

$$\lim_{h\to 0} \frac{\sqrt[3]{\frac{1}{64+h}} - \frac{1}{4}}{h} = \lim_{h\to 0} \frac{\sqrt[3]{(64+h)^{-1}} - 4^{-1}}{h}$$

$$= \lim_{h\to 0} \frac{(64+h)^{-\frac{1}{3}} - 64^{-\frac{1}{3}}}{h}$$

$$= \frac{d}{dx}\left(x^{-\frac{1}{3}}\right)\Big]_{x=64}$$

$$= -\frac{1}{3} x^{-\frac{4}{3}}\Big]_{x=64}$$

$$= -\frac{1}{3}(64)^{-\frac{4}{3}}$$

$$= -\frac{1}{3} \cdot \frac{1}{4^4}$$

$$= \frac{-1}{3\cdot 256}$$

$$= -\frac{1}{768}$$

Answer: $-\frac{1}{768}$

7. [8 points] For the function $h(x) = \frac{|x-8|}{x-8}$, find $\lim_{x\to 8^+} h(x)$ and $\lim_{x\to 8^-} h(x)$. Use these to compute $\lim_{x\to 8} h(x)$, or explain why the limit does not exist.

$$\lim_{x\to 8^+} \frac{|x-8|}{x-8} = \lim_{x\to 8^+} \frac{x-8}{x-8} = \lim_{x\to 8^+} 1 = 1$$

$$\lim_{x\to 8^-} \frac{|x-8|}{x-8} = \lim_{x\to 8^-} \frac{-(x-8)}{x-8} = \lim_{x\to 8^-} -1 = -1$$

These are unequal, so $\lim_{x\to 8} h(x)$ DNE

$\lim_{x\to 8^+} h(x)$ $= 1$

$\lim_{x\to 8^-} h(x)$ -1

$\lim_{x\to 8} h(x)$ DNE as the left and right limits are unequal.

8. [8 points] The position of a particle after t seconds is given by the equation $s(t) = e^{-t}(t^2 + t - 5)$. When is the (instantaneous) velocity 0?

   [You may assume $t \geq 0$]

$$s(t) = \frac{t^2+t-5}{e^t}$$

$$v(t) = s'(t) = \frac{[t^2+t-5]'e^t - (t^2+t-5)[e^t]'}{[e^t]^2}$$

$$= \frac{[2t+1]e^t - (t^2+t-5)e^t}{[e^t]^2}$$

$$= \frac{2t+1-t^2-t+5}{e^t}$$

$$= \frac{-t^2+t+6}{e^t} \overset{\text{set}}{=} 0$$

$$\Rightarrow -t^2+t+6=0$$

$$\Rightarrow t = \frac{-1 \pm \sqrt{1-4(1)(6)}}{2(-1)} = \frac{-1 \pm \sqrt{25}}{-2} = \frac{-1 \pm 5}{-2}$$

$$\Rightarrow t = \frac{-1-5}{-2}, \frac{-1+5}{-2} = 3, -2$$

$$\overset{t \geq 0}{\Longrightarrow} t = 3 \text{ seconds}$$

Answer: After 3 seconds

9. [8 points] Identify the locations of all vertical and horizontal asymptotes for the function below:

$$r(t) = \frac{3t^3 + 18t^2 + 15t}{8t^3 - 64t^2 + 56t}$$

HAs:

$$\lim_{t\to\infty} r(t) = \lim_{t\to\infty} \frac{t^3\left(3 + \frac{18}{t} + \frac{15}{t^2}\right)}{t^3\left(8 - \frac{64}{t} + \frac{56}{t^2}\right)} = \lim_{t\to\infty} \frac{3 + \frac{18}{t} + \frac{15}{t^2}}{8 - \frac{64}{t} + \frac{56}{t^2}}$$

$$= \frac{3+0+0}{8-0+0} = \frac{3}{8}$$

For similiar reasons, $\lim_{t\to-\infty} r(t) = \frac{3}{8}$

VAs: $r(t) = \frac{3t^3+18t^2+15t}{7t^3-56t^2+49t} = \frac{3t(t+5)(t+1)}{7t(t-7)(t-1)}$

Candiates for VAs occur when

$7t = 0$ OR $t-7=0$ OR $t-1=0$

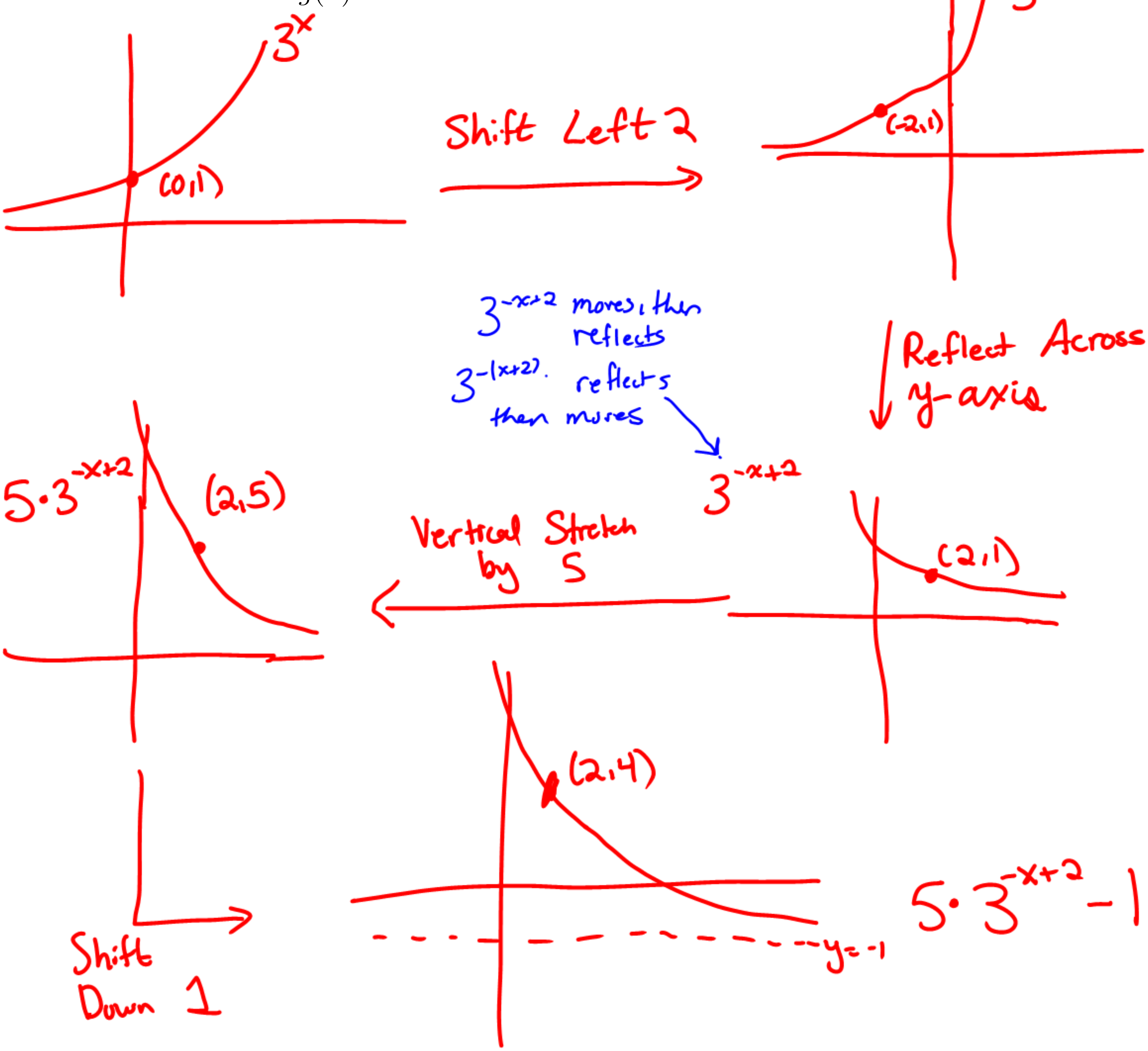
$t=0$ $\quad \boxed{t=7} \quad \boxed{t=1}$

Numerator is also 0 at 0, there is a hole not VA

Horizontal Asymptotes: $y = \frac{3}{8}$

Vertical Asymptotes: $t = 1, 7$

10. [8 points] Let $g(x)$ be the function whose graph is obtained by shifting the graph of 3^x 2 units to the left, reflecting it across the $y-$axis, stretching it vertically by a factor of 5, and then shifting it 1 unit down. What is the formula for $g(x)$?



$g(x) = 5\cdot 3^{-x+2} - 1$

11. [8 points] Find a formula for the inverse of the function:

$$u(x) = \frac{3x+5}{7x-1}$$

$$y = \frac{3x+5}{7x-1}$$

$$x = \frac{3y+5}{7y-1}$$

$$x(7y-1) = 3y+5$$

$$7xy - x = 3y+5$$

$$7xy - 3y = x+5$$

$$y(7x-3) = x+5$$

$$y = \frac{x+5}{7x-3}$$

$$u^{-1}(x) = \frac{x+5}{7x-3}$$

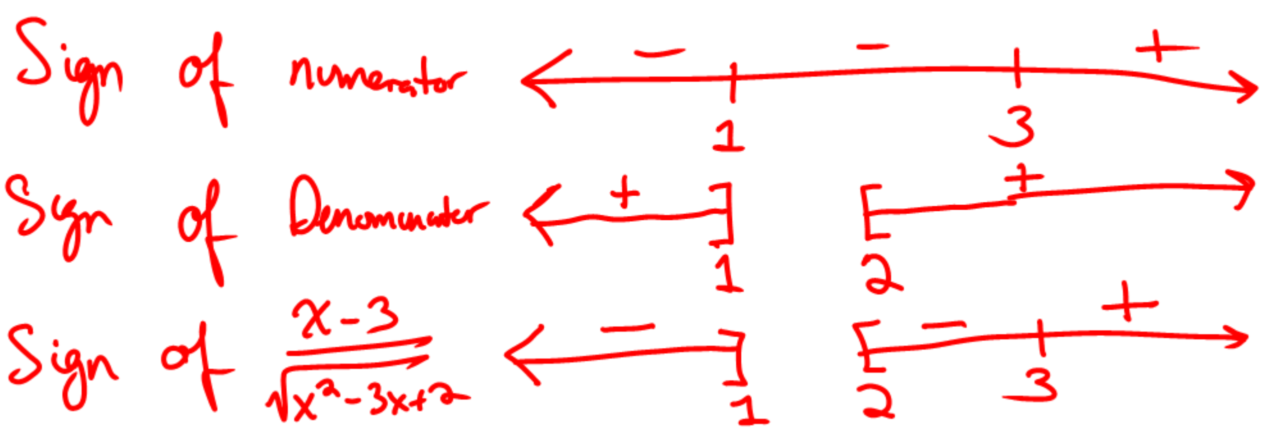
$u^{-1}(x) =$ $\frac{x+5}{7x-3}$

12. [8 points] Determine the limit:

$$\lim_{x \to 1^-} \frac{x-3}{\sqrt{x^2-3x+2}}$$

$\dfrac{x-3}{\sqrt{x^2-3x+2}} = \dfrac{x-3}{\sqrt{(x-1)(x-2)}}$, plugging in 1 we get

$\dfrac{\text{[Something Non-Zero]}}{0}$, so it is either ∞, $-\infty$, or DNE.



As $x \to 1^-$, the function is negative, so it tends to $-\infty$.

Answer: $-\infty$