MA 161 ONLINE, SUMMER 2026
Quiz 7: §4.1, 4.2, 4.3

Name: Key
PUID:

Score: /20
Length: 15 minutes

Directions: Follow these instructions carefully. Failure to do so may result in your quiz being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.

- Complete each question below. There are 2 questions; the maximum score is 20 points.
- You have 25 minutes: 15 minutes to complete the quiz and 10 minutes to upload it to Gradescope.
- This quiz is closed book and closed notes. Calculators are not allowed.
- You are to work independently. External help (including, but not limited to, using the internet and generative AI) is prohibited.
- You may use your own paper to complete this quiz; you do not need to print this document.
- Adequate work must be shown for each problem. A question with little to no work, even with a correct answer, will result in a low score.
- Make sure your work is neat and legible, especially after it is scanned. Use proper notation, and clearly mark the question numbers and your final answer. Points may be taken off if part of your work is unreadable.

(The questions start on the next page, one problem per page)

1. [5 points] The function $p(x) = x^6 - x^5 + 3x^4 - 5x^3 + 2x^2 - 12x - 24$ is a polynomial with roots at -1 and 2. Show $p'(x)$ has a root on the interval $(-1, 2)$.

   [Hint: It is unwise to try and find the exact value of the root; check if p satisfies the conditions of the Mean Value Theorem.]

p is differentiable on $(-1, 2)$ and continuous on $[-1, 2]$. So, MVT applies; ie, there is a point c on the interval $(-1, 2)$ such that

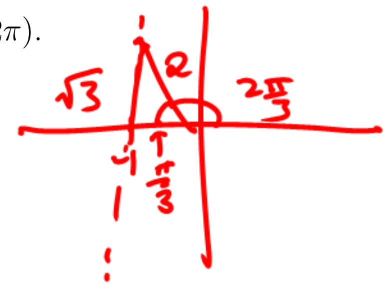
$$p'(c) = \frac{p(2) - p(-1)}{2 - (-1)} = \frac{0 - 0}{3} = 0$$

That c is a root of p'.

2. Consider the function $f(x) = x + 2\sin x$ on the interval $(0, 2\pi)$:

(a) [5 points] Determine all critical points of $f(x)$ on $(0, 2\pi)$.

$$f'(x) = 1 + 2\cos x \overset{\text{set}}{=} 0$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$
$$\text{if } x \in (0, 2\pi)$$



(b) [5 points] Determine the locations (i.e., the $x-$values) and classify all local extrema on $(0, 2\pi)$. Explain your reasoning.

We'll use the second derivative test

$$f''(x) = -2\sin x$$
$$f''\left(\frac{2\pi}{3}\right) = -2\underbrace{\sin\left(\frac{2\pi}{3}\right)}_{>0} < 0 \Rightarrow \text{Local max at } x = 2\pi/3$$
$$f''\left(\frac{4\pi}{3}\right) = -2\underbrace{\sin\left(\frac{4\pi}{3}\right)}_{<0} > 0 \Rightarrow \text{Local Min at } x = 4\pi/3$$

(c) [5 points] $f(x)$ has only 1 inflection point on $(0, 2\pi)$, where is it?

$$f''(x) = -2\sin x \overset{\text{set}}{=} 0$$
$$x = \pi \text{ if } x \in (0, 2\pi)$$

Technically we need to check f changes concavity at $x=\pi$, but since we know there is only one inflection point it has to be at $x=\pi$ by Fermat.

I didn't want them checking it during the quiz, hence why the question is worded the way it is.