MT 1 Review

Quiz 3

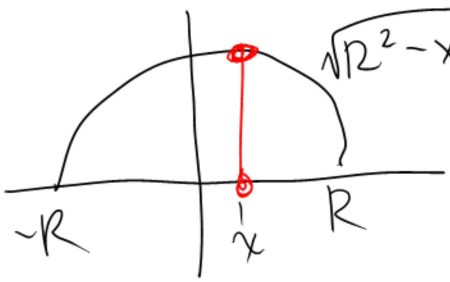
$$\frac{d}{dx}\left(\sin^2 x\right) = \underbrace{2\sin x \cos x = \sin(2x)}$$

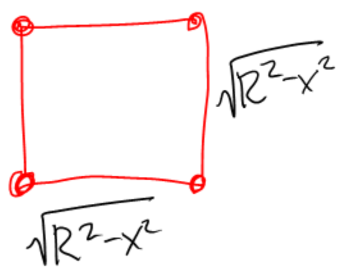
$[\sin x]^2$

$$\int \sin \pi x \, dx = -\frac{1}{\pi}\cos(\pi x) + C$$

Base (Semicircle of Radius R)

Cross Section: Square





$$A(x) = \left(\sqrt{R^2-x^2}\right)^2 = R^2 - x^2$$

$$V = \int_{-R}^{R} (R^2 - x^2)\,dx$$

Ex/ Find all possible anti-derivatives of $f(x) = \sin^3 x \cos^2 x$

$$\int \sin^3 x \cos^2 x \, dx = \int \sin^2 x \cos^2 x \sin x \, dx$$

$$= \int (1-\cos^2 x)\cos^2 x \sin x \, dx$$

$$= (-1)\int \left(\cos^2 x - \cos^4 x\right)(-1)\sin x \, dx$$

$$= -\left[\frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x\right] + C$$

$$= -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$$

$\int \sqrt{a^2 - x^2}\,dx$

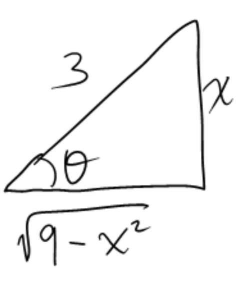
$x = a\sin\theta$

Ex/ Compute $\int \sqrt{9-x^2}\,dx = \int 3\cos\theta \; 3\cos\theta \, d\theta$

$$= 9\int \cos^2\theta \, d\theta \leftarrow \text{Power Reduction Formula}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= 9\int \left[\frac{1}{2} + \frac{1}{2}\cos 2\theta\right] d\theta$$



$x = 3\sin\theta$

$\boxed{\sin\theta = \frac{x}{3}}$

$dx = 3\cos\theta \, d\theta$

$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

$\sqrt{9-x^2} = 3\cos\theta$

$$= \int \frac{9}{2}\,d\theta + \frac{1}{2}\cdot\frac{9}{2}\int \cos 2\theta\,(2)\,d\theta = \frac{9}{2}\theta + \frac{9}{4}\sin 2\theta + C_0$$

$u = 2\theta$

$$= \frac{9}{2}\theta + \underbrace{\frac{9}{2}\sin\theta\cos\theta}_{\text{Uses } \sin 2\theta = 2\sin\theta\cos\theta} + C_0 = \frac{9}{2}\left[\sin^{-1}\left(\frac{x}{3}\right)\right] + \frac{9}{2}\underbrace{\left(\frac{x}{3}\right)}_{\sin\theta}\underbrace{\left(\frac{\sqrt{9-x^2}}{3}\right)}_{\cos\theta} + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{x}{2}\sqrt{9-x^2} + C$$

$$\int \frac{3x}{\sqrt{9-x^2}}$$

Ex/ Given $\vec{u}, \vec{v} \in \mathbb{R}^3$, name a vector that is orthogonal to both $\vec{u}$ and $\vec{v}$? Ans $\vec{u} \times \vec{v}$. $\vec{u}\cdot(\vec{u}\times\vec{v}) = \vec{v}\cdot(\vec{u}\times\vec{v}) = 0$

Orthogonal ("normal") if $\vec{u}\cdot\vec{v} = 0$

Ex/ Determine the volume of the parallelepiped determined $A(0,0,0)$; $B(2,3,1)$; $C(1,-1,0)$; $D(7,3,2)$

$V = \left|\vec{AB}\cdot(\vec{AC}\times\vec{AD})\right|$; $\vec{AB} = \langle 2,3,1\rangle$, $\vec{AC} = \langle 1,-1,0\rangle$, $\vec{AD} = \langle 7,3,2\rangle$; $V = \left|\begin{matrix} 2 & 3 & 1 \\ 1 & -1 & 0 \\ 7 & 3 & 2 \end{matrix}\right|$

↑ absolute value

$$\vec{AC}\times\vec{AD} = \left|\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 0 \\ 7 & 3 & 2 \end{matrix}\right| = \left|\begin{matrix} -1 & 0 \\ 3 & 2 \end{matrix}\right|\vec{i} - \left|\begin{matrix} 1 & 0 \\ 7 & 2 \end{matrix}\right|\vec{j} + \left|\begin{matrix} 1 & -1 \\ 7 & 3 \end{matrix}\right|k$$

$$= (-2-0)\vec{i} - (2)\vec{j} + (3-(-7))k$$

$$= -2\vec{i} - 2\vec{j} + 10\vec{k}$$

$\langle 2,3,1\rangle$

$$\vec{AB}\cdot(\vec{AC}\times\vec{AD}) = \langle 2,3,1\rangle\cdot\langle -2,-2,10\rangle = -4-6+10 = 0$$

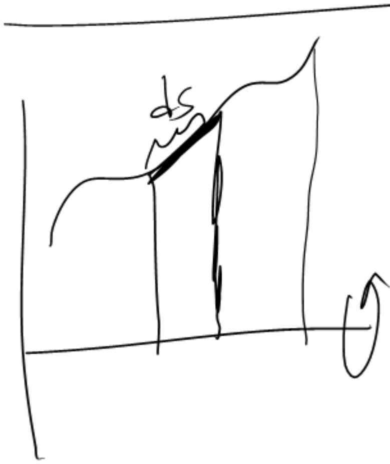
$W = \int F(x)\,dx$

EX/ We have a spring obeying Hooke's Law, find the work done by moving it from equilibrium to the point $x = a$

$$W = \int_0^a kx\,dx = k\int_0^a x\,dx = \frac{k}{2}\cdot x^2\Big]_0^a = \frac{1}{2}ka^2$$

EX/ In the previous problem it took 35 N to keep a spring 0.7 m stretched past equilibrium, find the spring constant.

$$F(x) = kx$$
$$35 = 0.7k$$
$$k = 50\ \frac{N}{m}$$



$$2\pi f(x)\,ds \qquad S = \int 2\pi f(x)\,\underbrace{ds}$$

$$ds = \underbrace{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}\,dx$$

$$\mathcal{L} = \int \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$
