Quiz 10: Taylor Series, Polar Coordinates (§11.4, 12.2 (Part I))

Score: /10

Name:____ Length: 15 minutes

Directions: Attempt all questions; you must show work for full credit. Use proper notation. In your work, clearly label question numbers and your final answer.

1. (3 points) Use the fact that $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ to show:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

> **Solution:**
>
> $$\begin{aligned} \lim_{x\to 0} \frac{\sin x}{x} &= \lim_{x\to 0} \frac{1}{x} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = \lim_{x\to 0} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n} \\ &\overset{Continuity}{=} 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} (\lim_{x\to 0} x)^{2n} \\ &= 1 \end{aligned}$$

2. (3 points) Convert the Cartesian coordinate $(2\sqrt{3}, -2)$ into polar coordinates. Express your final answer as a point (r, θ) where $r > 0$ and $\theta \in [-\pi, \pi)$.

> **Solution:** From the conversion equations
>
> $$\begin{aligned} r^2 &= x^2 + y^2 = (2\sqrt{3})^2 + (-2)^2 = 16 \\ \tan\theta &= \frac{y}{x} = \frac{-2}{2\sqrt{3}} = -\frac{1}{\sqrt{3}} \end{aligned}$$
>
> Choosing the positive root, $r = 4$. Since we are in the 4th quadrant $\theta = -\frac{\pi}{6} + 2\pi n$ (n is an integer). To get θ is the desired range, let $n = 0$; therefore, our point in polar coordinates is $(r, \theta) = (4, -\frac{\pi}{6})$.

3. (3 points) The polar curve $r = 6\sin\theta$ describes a circle, what is its center and radius? (Converting it into Cartesian is helpful, but not necessary)

> **Solution:** The equation $r = 2a\sin\theta$ describes a circle centered at $(0, a)$ with a radius of $|a|$. So, our circle is centered at $(0, 3)$ with a radius of 3.

4. (1 point) Final question of the semester: What's $2 + 2$?

> **Solution:** 4... I should not have to explain this one.