**Quiz 1:** Vectors (§13.1-13.4)

Score: /10

Name:____________________________________ Length: 15 minutes

**Directions:** Show work for full credit. Use proper notation. Clearly label final answers. You may use the back on this page if you need extra space.

1. For all parts, let $\vec{u} = \langle -1, 2, 3\rangle$ and $\vec{v} = \langle 2, 1, 1\rangle$.

   (a) (1 point) Compute $2\vec{u} + \vec{v}$.

   **Solution:** $2\vec{u} + \vec{v} = \langle 2(-1) + 2, 2(2) + 1, 2(3) + 1\rangle = \boxed{\langle 0, 5, 7\rangle}$

   (b) (1 point) Find the unit vector in the direction of $\vec{u}$.

   **Solution:** $|\vec{u}| = \sqrt{(-1)^2 + 2^2 + 3^2} = \sqrt{14}$. So, if $\hat{u}$ denotes the unit vector in the direction of $\vec{u}$.
   $$\hat{u} = \frac{1}{\sqrt{14}}\vec{u} = \langle \frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle$$

   (c) (4 points) Write $\vec{u}$ as the sum of two vectors, one parallel to $\vec{v}$ and one orthogonal to $\vec{v}$.

   **Solution:**
   $$\vec{u}^{\|} = \text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \frac{3}{6}\langle 2, 1, 1\rangle = \langle 1, \frac{1}{2}, \frac{1}{2}\rangle$$
   $$\vec{u}^{\perp} = \vec{u} - \text{proj}_{\vec{v}}\vec{u} = \langle -1, 2, 3\rangle - \langle 1, \frac{1}{2}, \frac{1}{2}\rangle = \langle -2, \frac{3}{2}, \frac{5}{2}\rangle$$
   $\vec{u} = \vec{u}^{\|} + \vec{u}^{\perp}$ where $\vec{u}^{\|}$ is parallel to $\vec{v}$ and $\vec{u}^{\perp}$ is orthogonal to $\vec{v}$.

   (d) (4 points) Find the area of the parallelogram with adjacent sides $\vec{u}$ and $\vec{v}$ (use units$^2$ as your unit).

   **Solution:**
   $$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix} = (-1)\vec{i} + 7\vec{j} - 5\vec{k}$$
   The area of the parallelogram is
   $$|\vec{u}\times\vec{v}| = \sqrt{(-1)^2 + 7^2 + (-5)^2} = \sqrt{75} = 5\sqrt{3} \text{ units}^2$$