Quiz 2: Solids of Revolution (§6.2-6.4)

Score: /10

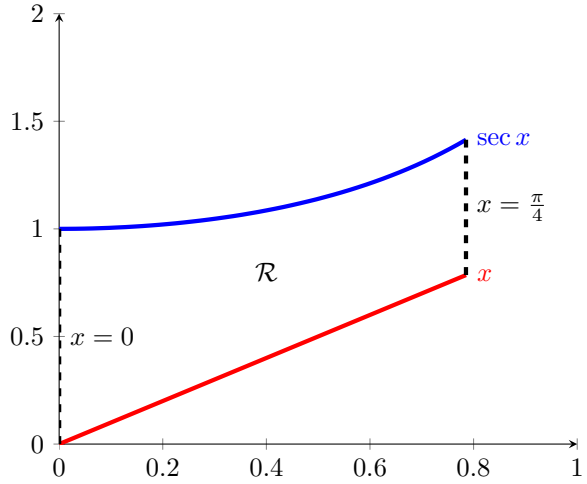
Name:_____ Length: 15 minutes

Directions: Answer all questions below; you must show work for full credit. Use proper notation. Clearly label final answers. You may use the back of this page if you need extra space.

1. For all parts, let $\mathcal{R}$ be the region bounded by the specified curves. Find the volume of the solid obtained by revolving $\mathcal{R}$ about the specified axis (you may use any method). Drawing a typical disk/washer/shell is highly recommended.

   (a) (4 points) $\mathcal{R}$ is the region bounded by $y = \sec x, y = x, x = 0$,and $x = \frac{\pi}{4}$. About the x-axis.

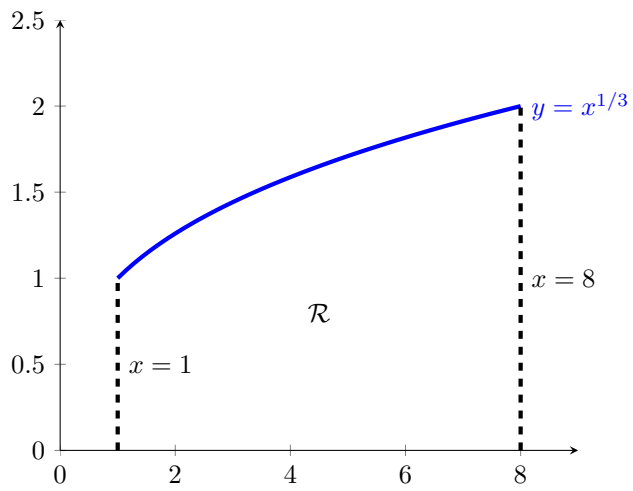
Solution: Here the washer method is used.



$$V = \int_0^{\pi/4} \pi(\sec^2 x - x^2)\, dx = \pi \left[\tan x - \frac{1}{3}x^3\right]_0^{\pi/4} = \pi\left(1 - \frac{\pi^3}{192}\right)$$

   (b) (3 points) $\mathcal{R}$ is the region bounded by $y = x^{1/3}, x = 1, x = 8$, and the x-axis. About the y-axis.

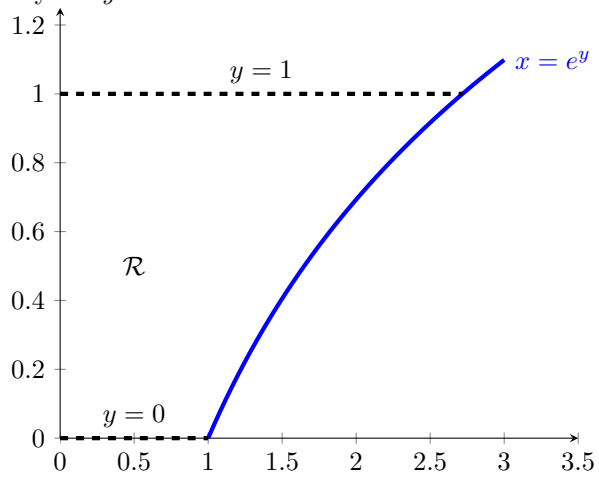
Solution: Here the shell method is used.



$$V = \int_1^8 2\pi x \cdot x^{1/3}\, dx = 2\pi \int_1^3 x^{4/3}\, dx = 2\pi \left[\frac{3}{7}x^{7/3}\right]_1^8 = 2\pi \left[\frac{3 \cdot 2^7}{7} - \frac{3}{7}\right] = \frac{762\pi}{7}$$

(c) (3 points) $\mathcal{R}$ is the region bounded by $y = \ln x, y = 0, y = 1$, and the y-axis. About the y-axis.

Solution: Here we want to use the disk method, but to do so we need to integrate with respect to y. So $y = \ln x$ becomes $x = e^y$



$$V = \int_0^1 \pi[e^y]^2\, dy = \frac{\pi}{2}[e^{2y}]_1^2 = \frac{\pi}{2}[e^2 - 1]$$