Quiz 4: Integration Techniques, Part I (§8.1-8.4)

Score: /10

Name:______________________________ Length: 15 minutes

Directions: Answer all the questions below in the space provided; you must show the work for full credit. Use proper notation. Clearly label the final answers.

1. For each part, compute the indefinite integral; you may use any (valid) method. Do not forget the constant of integration.

   (a) (3 points)

$$\int \frac{\ln x}{x^2}\ dx$$

Solution:
Use integration by parts with $u = \ln x$ and $dv = (1/x^2)dx$:

$$\begin{aligned}\int \frac{\ln x}{x^2} dx &= (\ln x)\left(-\frac{1}{x}\right) - \int -\frac{1}{x}\cdot\frac{1}{x}\ dx = -\frac{\ln x}{x} + \int \frac{1}{x^2} dx \\ &= -\frac{\ln x}{x} - \frac{1}{x} + C; \quad C \in \mathbb{R} \\ &= -\frac{\ln x + 1}{x} + C\end{aligned}$$

   (b) (3 points)

$$\int (1+\cos\theta)^2\ d\theta$$

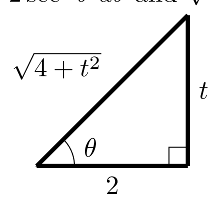
Solution: We will eventually need the fact that $\cos^2\theta = \frac{1}{2}[1+\cos 2\theta]$.

$$\begin{aligned}\int (1+\cos\theta)^2 d\theta &= \int (1 + 2\cos\theta + \cos^2\theta)\ d\theta = \theta + 2\sin\theta + \int \cos^2\theta\ d\theta \\ &= \theta + 2\sin\theta + \int \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right)\ d\theta \\ &= \theta + 2\sin\theta + \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C; \quad C \in \mathbb{R} \\ &= \frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta + C\end{aligned}$$

(c) (4 points)

$$\int \frac{t^3}{\sqrt{4+t^2}}\, dt$$

Solution: Let $t = 2\tan\theta$. Then $dt = 2\sec^2\theta\ d\theta$ and $\sqrt{4+t^2} = 2\sec\theta$.



$$\begin{aligned}
\int \frac{t^3}{\sqrt{4+t^2}}\, dt &= \int \frac{(2\tan\theta)^3}{2\sec\theta} \cdot 2\sec^2\theta\ d\theta = \int 8\tan^3\theta\sec\theta\ d\theta = 8\int \tan^2\theta(\sec\theta\tan\theta)\ d\theta \\
&= 8\int (\sec^2\theta - 1)(\sec\theta\tan\theta)\ d\theta \\
&= 8\left[\frac{1}{3}\sec^3\theta - \sec\theta\right] + C_0; \quad C_0 \in \mathbb{R} \\
&= \frac{8}{3}\sec^3\theta - 8\sec\theta + C_0 \\
&= \frac{8}{3}\left(\frac{\sqrt{4+t^2}}{2}\right)^3 + 8\left(\frac{\sqrt{4+t^2}}{2}\right) + C; \quad C \in \mathbb{R} \\
&= \frac{1}{3}(4+t^2)^{3/2} - 4\sqrt{4+t^2} + C
\end{aligned}$$

NOTE: This problem does not require trig sub. You can just make the substitution $u = 4 + t^2$ (can you see why?)