MA 261 ONLINE, SUMMER 2026
MIDTERM EXAM 1

Name (Print): Key ______________________ PUID: ______________________

Section Number: ______________________ Name of TA: ______________________

Start Time: ______________________ End Time: ______________________

Directions:

- Please use a **pen** if possible. Marks made by pen scan better than marks made by pencil.
- This is a **one-hour** exam.
- There are **12 questions** worth 8 points each; the maximum score is 100 points.
- The remaining 4 points are earned for filling out the information at the top of the page correctly. Section numbers, and their associated TA, are provided in the table below.

900	Eric Javier Pabon Cancel	**905**	Harrison Osprey Lemley
901	Jose Manuel Barrientos Lopez	**906**	Tyler Wallace Bruhnke
902	Leo Shen	**907**	Giovani Chuc Thai
903	Nick Polychronidis	**908**	Benjamin Scott Marlin
904	Matthew Charles Wackerfuss	**909**	Risa Michelle Fines

- Circle one and only one choice for each multiple-choice problem (Problems 1-3). **No partial credit** will be given on multiple-choice problems.
- Show all relevant work on each free-response problem (Problems 4-12). You must include some work or justification to receive full credit on these problems; partial credit may be given for steps leading to the correct solution. **Write the final answer in the box(es) provided**.
- Any external help or communication is not allowed, except with the proctor for logistical reasons. This exam is closed book and closed notes. **Calculators are not allowed**. Electronic devices, including smart watches, are to be turned off and put away.
- The last sheet is scratch paper. You may use only the scratch paper provided in this exam booklet. No additional scratch paper is permitted.

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

I agree to abide by the instructions above:

Student Signature: ______________________

Proctor Signature: ______________________

Question	Points	Score
1	8	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	8	
12	8	
Correct Header Info	4	
Total:	100	

1. [8 points] Identify the surface given by the equation

$$x^2 - y^2 - z^2 + 6y = 10$$

A. Elliptic Cylinder

B. Hyperboloid of One Sheet

C. Elliptic Cone

D. Hyperboloid of Two Sheets

E. Hyperbolic Paraboloid

$$x^2 - (y^2 - 6y + 9 - 9) - z^2 + 6y = 10$$

$$x^2 - (y-3)^2 + 9 - z^2 = 10$$

$$x^2 - (y-3)^2 - z^2 = 1$$

xy-trace: hyperbola

xz-trace: hyperbola

yz-trace: DNE

2. [8 points] Given $\vec{r}(t) = (t\sqrt{2})\vec{i} + e^t\vec{j} + e^{-t}\vec{k}$, find $\kappa(0)$, the curvature of $\vec{r}$ at $t = 0$.

   A. $\frac{1}{4}$

   B. $\frac{\sqrt{2}}{4}$

   C. $\frac{\sqrt{2}}{2}$

   D. 1

   E. $\sqrt{2}$

$\vec{v}(t) = \langle \sqrt{2}, e^t, -e^{-t} \rangle$

$\vec{a}(t) = \langle 0, e^t, e^t \rangle$

$\vec{v}(0) = \langle \sqrt{2}, 1, -1 \rangle$

$\vec{a}(0) = \langle 0, 1, 1 \rangle$

$$\vec{v}(0) \times \vec{a}(0) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \sqrt{2} & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 2\vec{i} - \sqrt{2}\vec{j} + \sqrt{2}\vec{k}$$

$$|\vec{v}(0) \times \vec{a}(0)| = \sqrt{4+2+2} = \sqrt{8}$$

$$|\vec{v}(0)|^3 = \left[\sqrt{2+1+1}\right]^3 = 2^3 = 8$$

$$\Rightarrow \kappa(0) = \frac{|\vec{v}(0) \times \vec{a}(0)|}{|\vec{v}(0)|^3} = \frac{\sqrt{8}}{8} = \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$$

3. [8 points] For $\vec{u} = \langle 1, 1, -5\rangle$ and $\vec{v} = \langle 1, 0, -1\rangle$, find a **positive** constant c where the area of the parallelogram with adjacent sides $\vec{u}$ and $c\vec{v}$ is 1.

  A. $\frac{\sqrt{2}}{6}$

  B. $\frac{\sqrt{38}}{38}$

  C. $\frac{1}{18}$

  D. $3\sqrt{2}$

  E. $\frac{1}{6}$

$c\vec{v} = \langle c, 0, -c\rangle$

$$\vec{u} \times c\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -5 \\ c & 0 & -c \end{vmatrix} = -c\vec{i} - (-c + 5c)\vec{j} - c\vec{k}$$

$$= c\left[-\vec{i} - 4\vec{j} - \vec{k}\right]$$

Area of Parallelogram $= |\vec{u} \times c\vec{v}|$

$$= \sqrt{c^2 + 16c^2 + c^2}$$

$$= |c|\sqrt{18} \overset{\text{set}}{=} 1$$

$$|c| = \frac{1}{\sqrt{18}}$$

$\overset{c>0}{\Longrightarrow}$ $c = \frac{1}{\sqrt{18}}$

$$= \frac{\sqrt{18}}{18}$$

$$= \frac{3\sqrt{2}}{18}$$

$$c = \frac{\sqrt{2}}{6}$$

4. [8 points] Find a parametric equation for the line that lies at the intersection of the planes:

$$\begin{cases} 2x + 3y + z = 6 \\ 3x - 2y - z = -4 \end{cases}$$

Let $t \in \mathbb{R}$ and set $z = t$:

$$\begin{cases} 2x + 3y + t = 6 \\ 3x - 2y - t = -4 \end{cases} \Rightarrow \begin{cases} 2x + 3y = 6 - t \\ 3x - 2y = -4 + t \end{cases} \Rightarrow \begin{cases} 6x + 9y = 18 - 3t \\ 6x - 4y = -8 + 2t \end{cases}$$

Subtract both equations

$$13y = 26 - 5t$$

$$y = 2 - \frac{5}{13}t$$

Now,

$$2x + 3y + t = 6$$

$$2x + 3\left(2 - \frac{5}{13}t\right) + t = 6$$

$$2x + 6 - \frac{15}{13}t + t = 6$$

$$2x - \frac{2}{13}t = 0$$

$$2x = \frac{2}{13}t$$

$$x = \frac{1}{13}t$$

$\Rightarrow$ An equation is $\begin{cases} x = \frac{1}{13}t \\ y = 2 - \frac{5}{13}t \\ z = t \end{cases}$ for $-\infty < t < \infty$. Scale t to $13t$ to get another one

Equation: $\begin{cases} x = t \\ y = 2 - 5t \\ z = 13t \end{cases}$; $-\infty < t < \infty$

5. [8 points] Find a parametrization for the ellipse $\frac{x^2}{36} + \frac{y^2}{64} = 1$. Use it to set up an integral that computes its perimeter; simplify your answer, but **do not evaluate** the integral.

Parametrization: $\vec{r}(t) = \langle 6\cos t, 8\sin t \rangle$; $0 \leq t \leq 2\pi$

$\vec{r}'(t) = \langle -6\sin t, 8\cos t \rangle$

$|\vec{r}'(t)| = \sqrt{(-6\sin t)^2 + (8\cos t)^2} = \sqrt{36\sin^2 t + 64\cos^2 t}$

Perimeter = Arc Length

$= \int_0^{2\pi} |r'(t)|\, dt$

$= \int_0^{2\pi} \sqrt{36\sin^2 t + 64\cos^2 t}\, dt$

Perimeter= $$\int_0^{2\pi} \sqrt{36\sin^2 t + 64\cos^2 t}\; dt$$

6. [8 points] What are the partial derivatives of $f(x, y) = e^{xy}$ after a switch to polar coordinates? That is, if $x = r\cos\theta$ and $y = r\sin\theta$, use the Chain Rule to compute f_r and f_θ. For full points, write f_r and f_θ as expressions of only r and θ.

By the Chain Rule,

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r}$$

$$= \left[y e^{xy}\right]\left[\cos\theta\right] + \left[x e^{xy}\right]\left[\sin\theta\right]$$

$$= \left[r\sin\theta\, e^{r^2\sin\theta\cos\theta}\right]\left[\cos\theta\right] + \left[r\cos\theta\, e^{r^2\sin\theta\cos\theta}\right]\left[\sin\theta\right]$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta}$$

$$= \left[y e^{xy}\right]\left[-r\sin\theta\right] + \left[x e^{xy}\right]\left[r\cos\theta\right]$$

$$= \left[r\sin\theta\, e^{r^2\sin\theta\cos\theta}\right]\left[-r\sin\theta\right] + \left[r\cos\theta\, e^{r^2\sin\theta\cos\theta}\right]\left[r\cos\theta\right]$$

$f_r(r,\theta) =$ $r\sin\theta e^{(r\cos\theta)(r\sin\theta)}[\cos\theta] + r\cos\theta e^{(r\cos\theta)(r\sin\theta)}[\sin\theta] = 2r\sin\theta\cos\theta e^{r^2\sin\theta\cos\theta}$

$f_\theta(r,\theta) =$ $r\sin\theta e^{(r\cos\theta)(r\sin\theta)}[-r\sin\theta] + r\cos\theta e^{(r\cos\theta)(r\sin\theta)}[r\cos\theta]$

$= r^2(\cos^2\theta - \sin^2\theta)e^{r^2\sin\theta\cos\theta}$

7. [8 points] Find $\frac{\partial z}{\partial y}$, where:

$$\frac{1}{z} = \frac{y}{x} - \frac{x}{y}$$

implicitly defines z as a differentiable function of x and y.

$$F(x,y,z) = \frac{1}{z} - \frac{y}{x} + \frac{x}{y}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-\frac{1}{x} - \frac{x}{y^2}}{-\frac{1}{z^2}}$$

$$= z^2\left[-\frac{1}{x} - \frac{x}{y^2}\right]$$

$$= -z^2\left[\frac{1}{x} + \frac{x}{y^2}\right]$$

$\frac{\partial z}{\partial y} = -z^2\left[\frac{1}{x} + \frac{x}{y^2}\right]$

8. [8 points] A ball is thrown off a 20 m tall building eastward with an initial speed of 15 m/s. In addition to gravity, winds make the ball accelerate northward at a constant rate of $5\,\mathrm{m/s^2}$. If the acceleration due to gravity is $g \approx 10\,\mathrm{m/s^2}$ downward, how far from the base of the building does the ball land?
I.e., if the base of the building is at the origin, determine the distance between the origin and the ball's landing spot in the xy-plane.

① Find the position function given

$$\begin{cases} \vec{a}(t) = \langle 0, 5, -10 \rangle \\ \vec{v}(0) = \langle 15, 0, 0 \rangle \\ \vec{r}(0) = \langle 0, 0, 20 \rangle \end{cases}$$

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle 0, 5t, -10t \rangle + \vec{C}$

$\vec{v}(0) = \langle 0, 0, 0 \rangle + \vec{C} \overset{\text{set}}{=} \langle 15, 0, 0 \rangle$

$\vec{C} = \langle 15, 0, 0 \rangle$

$\Rightarrow \vec{v}(t) = \langle 15, 5t, -10t \rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 15t, \frac{5}{2}t^2, -5t^2 \rangle + \vec{C}$

$\vec{r}(0) = \langle 0, 0, 0 \rangle + \vec{C} \overset{\text{set}}{=} \langle 0, 0, 20 \rangle$

$\vec{C} = \langle 0, 0, 20 \rangle$

$\Rightarrow \vec{r}(t) = \langle 15t, \frac{5}{2}t^2, -5t^2 + 20 \rangle$

② Find Distance when z-component is 0:

$-5t^2 + 20 \overset{\text{set}}{=} 0$

$t^2 = 4 \leftarrow t$ is assumed to be ≥ 0

$t = 2$

x-comp: $x = 15(2) = 30$

y-comp: $y = \frac{5}{2}(2)^2 = 5 \cdot 2 = 10$

Distance $= \sqrt{x^2 + y^2} = \sqrt{(30)^2 + (10)^2} = \sqrt{900 + 100} = \sqrt{1000}$

$= 10\sqrt{10}$ meters

Distance: $10\sqrt{10}$ meters

9. [8 points] Find a constant a that would make $h(x, y)$ continuous at the point $(-1, 1)$.

$$h(x,y) = \begin{cases} \frac{x^2-y^2}{x^2-xy-2y^2} & (x,y) \neq (-1,1) \\ a & (x,y) = (-1,1) \end{cases}$$

a is the limit as $(x,y) \to (-1,1)$

$$a = \lim_{(x,y)\to(-1,1)} \frac{x^2-y^2}{x^2-xy-2y^2} = \lim_{(x,y)\to(-1,1)} \frac{(x+y)(x-y)}{(x+y)(x-2y)}$$

$$= \lim_{(x,y)\to(-1,1)} \frac{x-y}{x-2y} = \frac{-1-1}{-1-2(1)} = \frac{-2}{-3} = \frac{2}{3}$$

$a = \frac{2}{3}$

10. [8 points] If $g(x, y) = \ln(x^2 + y^2)$ and $\vec{u} = \langle 3, 4 \rangle$, find $D_{\vec{u}}g(1, 2)$, the directional derivative of g in the direction of $\vec{u}$ at the point $P(1, 2)$.

$$\nabla g = \left\langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \right\rangle$$

$$\nabla g(1,2) = \left\langle \frac{2}{5}, \frac{4}{5} \right\rangle$$

$$\hat{u} = \frac{\vec{u}}{|\vec{u}|} = \frac{1}{5}\langle 3,4 \rangle = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

$$D_{\vec{u}}\, g(1,2) = \nabla g(1,2) \cdot \hat{u} = \left\langle \frac{2}{5}, \frac{4}{5} \right\rangle \cdot \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

$$= \frac{6}{25} + \frac{16}{25}$$

$$= \frac{22}{25}$$

$D_{\vec{u}}g(1, 2) = \frac{22}{25}$

11. [8 points] Let Σ be the plane tangent to the surface $z = e^x + \frac{2}{y}$ at the point $P(0, 1, 3)$. Where does Σ intersect the x-axis?

① Find Σ

$F(x,y,z) = z - e^x - \frac{2}{y}$

$\nabla F = \langle -e^x, \frac{2}{y^2}, 1 \rangle$

$\nabla F(0,1,3) = \langle -1, 2, 1 \rangle$

Then Σ is the plane

$\nabla F(0,1,3) \cdot \langle x, y-1, z-3 \rangle = 0$

$\langle -1, 2, 1 \rangle \cdot \langle x, y-1, z-3 \rangle = 0$

$-x + 2(y-1) + (z-3) = 0$

$-x + 2y + z = 5$

② Find x-intercept

$-x + 0 + 0 = 5$

$x = -5$

Location of Intersection: (-5,0,0)

12. [8 points] If $u(x,t) = \sin(x-2t) + \cos(x+2t)$, compute the following expression:

$$\frac{\partial^2 u}{\partial t^2} - 4\frac{\partial^2 u}{\partial x^2}$$

$u_t = -2\cos(x-2t) - 2\sin(x+2t)$

$u_{tt} = -4\sin(x-2t) - 4\cos(x+2t)$

$u_x = \cos(x-2t) - \sin(x+2t)$

$u_{xx} = -\sin(x-2t) - \cos(x+2t)$

$-4u_{xx} = 4\sin(x-2t) + 4\cos(x+2t)$

these add to 0

$\frac{\partial^2 u}{\partial t^2} - 4\frac{\partial^2 u}{\partial x^2} =$ 0