MA 261 ONLINE, SUMMER 2026
FINAL EXAM

Name (Print): Key ______________ PUID: ______________

Section Number: ______________ Name of TA: ______________

Start Time: ______________ End Time: ______________

Directions:

- Please use a **pen** if possible. Marks made by pen scan better than marks made by pencil.
- This is a **two-hour** exam.
- There are **20 questions** worth 10 points each; the maximum score is 200 points.
- Fill out the information at the top of the page correctly. Section numbers, and their associated TA, are provided in the table below.

900	Eric Javier Pabon Cancel	**905**	Harrison Osprey Lemley
901	Jose Manuel Barrientos Lopez	**906**	Tyler Wallace Bruhnke
902	Leo Shen	**907**	Giovani Chuc Thai
903	Nick Polychronidis	**908**	Benjamin Scott Marlin
904	Matthew Charles Wackerfuss	**909**	Risa Michelle Fines

- Circle one and only one choice for each multiple-choice problem (Problems 1-6). **No partial credit** will be given on multiple-choice problems.
- Show all relevant work on each free-response problem (Problems 7-24). You must include some work or justification to receive full credit on these problems; partial credit may be given for steps leading to the correct solution. **Write the final answer in the box(es) provided**.
- Any external help or communication is not allowed, except with the proctor for logistical reasons. This exam is closed book and closed notes. **Calculators are not allowed**. Electronic devices, including smart watches, are to be turned off and put away.
- The last sheet is scratch paper. You may use only the scratch paper provided in this exam booklet. No additional scratch paper is permitted.

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

I agree to abide by the instructions above:

Student Signature: ______________________

Proctor Signature: ______________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
16	10	
17	10	
18	10	
19	10	
20	10	
Total:	200	

1. [10 points] For $\mathbf{u} = \langle 2, 3, -1\rangle$ and $\mathbf{v} = \langle 1, -1, 2\rangle$, find the area of the parallelogram with adjacent sides $\mathbf{u}$ and $\mathbf{v}$.

   A. $\frac{\sqrt{3}}{15}$

   B. $5\sqrt{3}$

   C. $\sqrt{59}$

   D. 75

   E. 3

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 5\vec{i} - 5\vec{j} - 5\vec{k}$$

$$\text{Area} = |\vec{u} \times \vec{v}| = \sqrt{75} = 5\sqrt{3}$$

2. [10 points] Recall from Quiz 8 that the divergence of a magnetic field is always 0. Which of the following vector fields can **NOT** be used to represent a magnetic field?

   A. $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} - (2xz + 2yz)\mathbf{k}$

   B. $\mathbf{F}(x, y) = (5 - x^2y)\mathbf{i} + xy^2\mathbf{j}$

   C. $\mathbf{F}(x, y, z) = \text{curl}(\langle 2x + 1, y, -z\rangle)$

   D. $\mathbf{F}(x, y, z) = x^2\mathbf{i} - 2xy\mathbf{j} + xz\mathbf{k}$

   E. $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$

$\text{div}\,\vec{F} = 2x - 2x + x = x \not\equiv 0$

The rest have divergence 0

3. [10 points] Let S be the triangular region $2x + 3y + z = 6$ in the first octant. Compute the flux

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

across S, where $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$ and $\mathbf{n}$ is an upward pointing normal vector.

A. -12

B. 0

C. 6

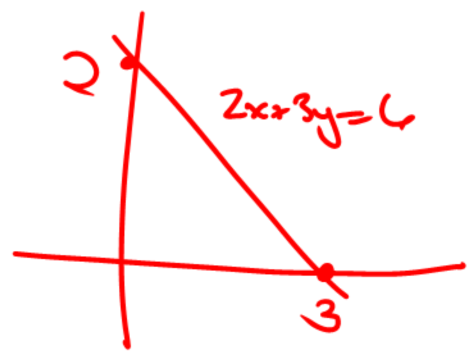
D. 18

E. 36

$\vec{r}(x,y) = \langle x, y, 6 - 2x - 3y \rangle$

$\vec{n} = \frac{\partial \vec{r}}{\partial x} \times \frac{\partial \vec{r}}{\partial y} = \begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 0 & 1 & -3 \end{vmatrix} = 2\vec{i} + 3\vec{j} + \vec{k} = \langle 2, 3, 1 \rangle$

$\iint F \cdot \vec{n}\, dS = \iint_{\text{Triangle}} \langle x, y, z \rangle \cdot \langle 2, 3, 1 \rangle\, dA$



$= \iint (2x + 3y + z)\, dA$

$= \iint 6\, dA$

$= 6 \cdot \frac{1}{2} \cdot 2 \cdot 3$

$= 18$

4. [10 points] Let M and m be the maximum and minimum of

$$f(x,y) = 2x^2 + 5y^2 + 3$$

on the closed disk $D = \{(x,y) \mid x^2 + y^2 \leq 4\}$. What is the value of $M + m$?

A. 3

B. 18

C. 20

D. 23

E. 26

Interior:

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \Rightarrow \begin{cases} 4x = 0 \\ 10y = 0 \end{cases} \Rightarrow (0,0) \text{ is a critical point}$$

Boundary $(x^2+y^2=4)$:

$$f(x,y) = 2x^2 + 5y^2 + 3$$

$$f(x) = 2x^2 + 5(4-x^2) + 3$$

$$= -3x^2 + 23$$

$$23 - 12 = 11$$

$$f'(x) = -6x = 0$$

$$x = 0$$

Values: Interior $f(0,0) = 3$

Bdy

x	-2	0	2
$f(x)$	11	23	11

$M = 23$

$m = 3$

$M + m = 26$

5. [10 points] Which of the following parametric surfaces describes the hemisphere $H = \{(x, y, z)|\ x^2 + y^2 + z^2 = 4,\ z \geq 0\}$? (Look carefully)

   A. $\mathbf{r}(u, v) = \langle 2\sin u \cos v,\ 2\sin u \sin v,\ 2\cos u\rangle;\quad 0 \leq u \leq \pi,\ 0 \leq v \leq \pi$

   B. $\mathbf{r}(u, v) = \langle 2\sin u \cos v,\ 2\sin u \sin v,\ 2\cos u\rangle;\quad 0 \leq u \leq \pi,\ 0 \leq v \leq 2\pi$

   C. $\mathbf{r}(u, v) = \langle 2\sin u \cos v,\ 2\sin u \sin v,\ 2\cos u\rangle;\quad 0 \leq u \leq \frac{\pi}{2},\ 0 \leq v \leq 2\pi$

   D. $\mathbf{r}(u, v) = \langle 4\sin u \cos v,\ 4\sin u \sin v,\ 4\cos u\rangle;\quad 0 \leq u \leq \frac{\pi}{2},\ 0 \leq v \leq 2\pi$

   E. $\mathbf{r}(u, v) = \langle 4\sin u \cos v,\ 4\sin u \sin v,\ 4\cos u\rangle;\quad 0 \leq u \leq \pi,\ 0 \leq v \leq 2\pi$

Spherical Coordinates with u instead of φ and v instead of θ

6. [10 points] Identify the surface given by the equation

$$x^2 - y^2 + 2x - z + 2 = 0$$

A. Elliptic Cylinder

B. Hyperbolic Paraboloid

C. Hyperboloid of One Sheet

D. Elliptic Cone

E. Hyperboloid of Two Sheets

$x^2 + 2x + 1 - 1 - y^2 - z + 2 = 0$

$(x+1)^2 - y^2 - z + 1 = 0$

7. [10 points] Calculate the outward flux of the vector field

$$\mathbf{F}(x,y,z) = \left\langle x,\ e^{\sin(x^2\ln(z+5))},\ z - \tan^{-1}(x^2e^y)\right\rangle$$

across the cube $Q = \{(x,y,z)|\ -1 \le x \le 1,\ -1 \le y \le 1,\ -1 \le z \le 1\}$.

$\operatorname{div} \vec{F} = 1 + 0 + 1 = 2$

$\text{Flux} \underset{\text{Thm}}{\overset{\text{Divergence}}{=}} \int_{\text{Cube}} 2\, dV = 2 \cdot 2^3 = 16$

Flux = 16

8. [10 points] Consider the surface S defined by the plane $z = x + y$ over the region $D = \{(x, y) \mid 0 \le y \le x,\ 0 \le x \le 1\}$. If the density of S at the point (x, y, z) is given by $\rho(x, y, z) = xy$, what is the mass of S?

$$\vec{r}(x,y) = \langle x, y, x+y \rangle$$

$$|\vec{r}_x \times \vec{r}_y| = |\langle -1, -1, 1 \rangle| = \sqrt{3}$$

$$\text{Mass} = \iint_S \rho \, dS = \int_0^1 \int_0^x xy\sqrt{3}\, dy\, dx$$

$$= \sqrt{3} \int_0^1 x \left[\tfrac{1}{2} y^2\right]_0^x dx$$

$$= \frac{\sqrt{3}}{2} \int_0^1 x^3 \, dx$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{4} x^4 \Big]_0^1$$

$$= \frac{\sqrt{3}}{8}$$

Mass= $\frac{\sqrt{3}}{8}$

9. [10 points] Let Σ be the plane tangent to the surface $x^2 + 2y^2 - 3z^2 - 3 = 0$ at the point $(2, -1, 1)$. Where does Σ intersect the x-axis?

$$F(x,y,z) = x^2 + 2y^2 - 3z^2 - 3$$

$$\nabla F = \langle 2x, 4y, -6z \rangle$$

$$\nabla F(2,-1,1) = \langle 4, -4, -6 \rangle$$

$$\Sigma: \langle 4, -4, -6 \rangle \cdot \langle x-2, y+1, z-1 \rangle = 0$$

$$4(x-2) - 4(y+1) - 6(z-1) = 0$$

Find x-intercept:

$$4(x-2) - 4 + 6 = 0$$

$$4x - 8 - 4 + 6 = 0$$

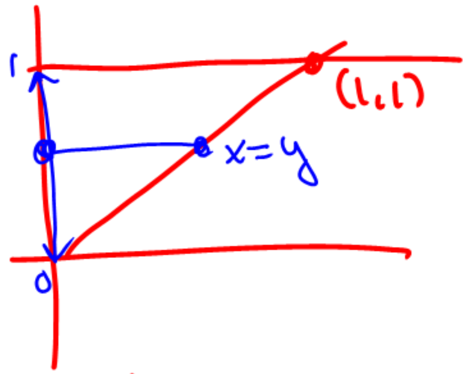
$$4x - 6 = 0$$

$$x = \frac{3}{2}$$

Location of Intersection: $(\frac{3}{2}, 0, 0)$

10. [10 points] Reverse the order of integration and evaluate:

$$\int_0^1 \int_x^1 \cos\left(\frac{\pi}{2}y^2\right)\ dydx$$



$$= \int_0^1 \int_0^y \cos\left(\frac{\pi}{2}y^2\right) dx\, dy$$

$$= \int_0^1 y \cos\left(\frac{\pi}{2}y^2\right) dy$$

$$= \frac{1}{\pi}\int_0^1 \cos\left(\frac{\pi}{2}y^2\right)(\pi y)\, dy$$

$$= \frac{1}{\pi} \sin\left(\frac{\pi}{2}y^2\right)\Big]_0^1$$

$$= \frac{1}{\pi}[1-0]$$

$$= \frac{1}{\pi}$$

$$\int_0^1 \int_x^1 \cos\left(\frac{\pi}{2}y^2\right)\ dydx = \frac{1}{\pi}$$

11. [10 points] Let E be the ball $x^2+y^2+z^2 \leq 4$. The density at the point (x,y,z) is $\delta(x,y,z) = k\sqrt{x^2+y^2+z^2}$ for some constant k. What value does k need to be in order for E to have a mass of 1?

[NOTE: Since you'll need spherical coordinates, we use δ to denote density as opposed to ρ.]

$$\text{Mass} = \int_{\text{Ball}} \delta \, dV$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{2} k\rho \cdot \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= (2\pi k)\left(\int_0^{\pi} \sin\varphi \, d\varphi\right)\left(\int_0^{2} \rho^3 \, d\rho\right)$$

$$= 2\pi k \left[-\cos\varphi\right]_0^{\pi} \cdot \left[\tfrac{1}{4}\rho^4\right]_0^{2}$$

$$= 2\pi k [2][4]$$

$$= 16\pi k \underset{=}{\overset{\text{set}}{}} 1$$

$$k = \frac{1}{16\pi}$$

$k = \frac{1}{16\pi}$

12. [10 points] If $\mathbf{F}(x, y, z) = \langle 2xy + z, x^2 + 2yz, y^2 + x\rangle$, find the curl of $\mathbf{F}$ at the point $(1, 2, 3)$.

$$\text{curl}\,\vec{F} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ 2xy+z & x^2+2yz & y^2+x \end{vmatrix}$$

$$= \langle 0, 0, 0\rangle$$

So, curl $\vec{F}$ at $(1,2,3)$ is $\langle 0,0,0\rangle$

curl($\mathbf{F}$) at $(1, 2, 3)$: $\langle 0, 0, 0\rangle$

13. [10 points] Use Stokes' Theorem to compute:

$$\iint_S \text{curl}(\mathbf{F}) \cdot \mathbf{n}\, dS$$

where $\mathbf{F}(x, y, z) = \langle -y, x, 0 \rangle$ and $S = \{(x, y, z) \mid x^2 + y^2 \leq 1,\ z = 0\}$ is the unit disk oriented with an upward facing normal vector.

The boundary of S is the unit circle [call it C].

Parametrize C:

$$\vec{r}(t) = \langle \cos t, \sin t \rangle; \quad 0 \leq t \leq 2\pi$$

$$\vec{r}'(t) = \langle -\sin t, \cos t \rangle$$

$$\vec{F}(x(t), y(t), z(t)) = \langle -\sin t, \cos t \rangle$$

$$\iint_S F \cdot \vec{n}\, dS \overset{\text{Stokes'}}{=} \int_C \vec{F} \cdot d\vec{r}$$

$$= \int_0^{2\pi} \langle -\sin t, \cos t \rangle \cdot \langle -\sin t, \cos t \rangle\, dt$$

$$= \int_0^{2\pi} (\sin^2 t + \cos^2 t)\, dt$$

$$= \int_0^{2\pi} dt = 2\pi$$

$\iint_S \text{curl}(\mathbf{F}) \cdot \mathbf{n}\, dS =$ 2π

14. [10 points] If $g(x,y) = \frac{x}{y}$ and $\mathbf{u} = \langle -1, 2 \rangle$, find $D_{\mathbf{u}}g(-3,1)$, the directional derivative of g in the direction of $\mathbf{u}$ at the point $(-3,1)$.

$$\nabla g = \left\langle \frac{1}{y}, -\frac{x}{y^2} \right\rangle$$

$$\nabla g(-3,1) = \langle 1, 3 \rangle$$

$$\hat{u} = \frac{1}{\sqrt{5}} \langle -1, 2 \rangle$$

$$D_{\vec{u}}\, g(-3,1) = \langle 1, 3 \rangle \cdot \frac{1}{\sqrt{5}} \langle -1, 2 \rangle$$

$$= \frac{1}{\sqrt{5}} [-1 + 6]$$

$$= \frac{5}{\sqrt{5}}$$

$$= \sqrt{5}$$

$D_{\mathbf{u}}g(-3,1) = \sqrt{5}$

15. [10 points] Let $a, b > 0$ be constants. A parameterization for the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is given by:

$$\vec{r}(t) = \langle a\cos t, b\sin t\rangle; \quad 0 \leq t \leq 2\pi$$

Use it to set up an integral that computes the perimeter of the ellipse. Simplify your answer, but **do not evaluate** the integral.

$\vec{r}\,'(t) = \langle -a \sin t, b\cos t \rangle$

Perimeter = Arc Length

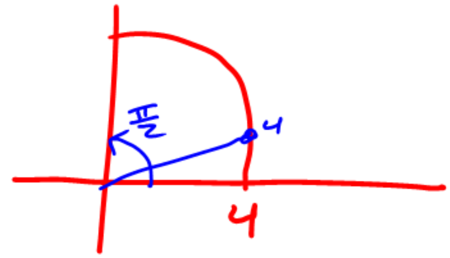
$= \int_0^{2\pi} |\vec{r}\,'(t)|\, dt$

$= \int_0^{2\pi} \sqrt{a^2\sin^2 t + b^2 \cos^2 t}\ dt$

Perimeter= $\int_0^{2\pi} \sqrt{a^2 \sin^2 t + b^2 \cos^2 t}\ dt$

16. [10 points] Find the average value of $f(x,y) = e^{-(x^2+y^2)}$ over the quarter-disk $x^2 + y^2 \leq 16$ with $x, y \geq 0$.

Area: $\frac{1}{4}\pi \cdot 4^2 = 4\pi$



Average: $\frac{1}{\text{Area}} \int f \, dA = \frac{1}{4\pi} \int e^{-(x^2+y^2)} \, dA$

$$= \frac{1}{4\pi} \int_0^{\pi/2} \int_0^4 e^{-r^2} r \, dr \, d\theta$$

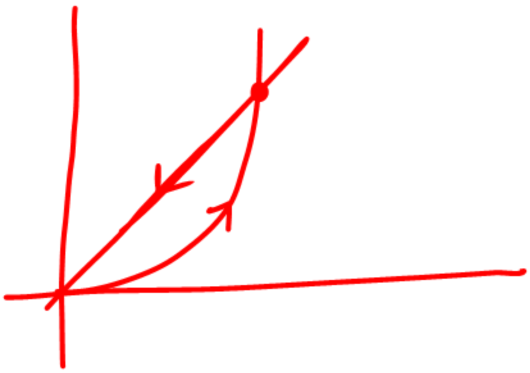
$$= \frac{\pi}{2 \cdot 4\pi} \int_0^4 e^{-r^2} r \, dr \qquad = -\frac{1}{2} \cdot \frac{1}{8} \int_0^4 e^{-r^2} [-2r] \, dr$$

$$= -\frac{1}{16} \cdot e^{-r^2} \Big]_0^4$$

$$= \frac{1}{16}\left[1 - e^{-16}\right]$$

Average= $\frac{1}{16}[1 - e^{-16}]$

17. [10 points] Let C be the positively oriented boundary of the region in $\mathbb{R}^2$ bounded by $y = x^2$ and $y = x$. Use the flux form of Green's Theorem to find the outward flux of $\mathbf{F}(x, y) = (4x - y^2)\mathbf{i} + (x^2 - y)\mathbf{j}$ across C.



$$\oint_C F\cdot n\, ds \overset{\text{Green's}}{=} \iint \frac{\partial}{\partial x}(4x-y^2)+\frac{\partial}{\partial y}(x^2-y)\,dA$$

$$= \iint (4-1)\,dA$$

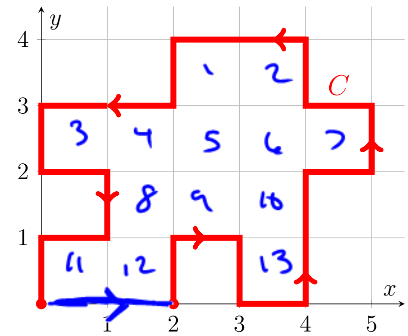
$$= 3\iint dA = 3\int_0^1\int_{x^2}^{x} dy\,dx$$

$$= 3\int_0^1 [x - x^2]\,dx = 3\left[\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]_0^1$$

$$= 3\left[\tfrac{1}{2} - \tfrac{1}{3}\right] = 3\cdot\tfrac{1}{6} = \frac{1}{2}$$

Flux= $\frac{1}{2}$

18. [10 points] The curve C depicted below travels from $(2,0)$ to $(0,0)$ in the xy-plane (where one grid square is 1 square unit of area). What is the value of $\int_C \mathbf{F}\cdot d\mathbf{r}$ if $\mathbf{F}(x,y) = \langle x+y^2,\ 3x+2xy\rangle$.



[**HINT:** If L is the line segment from $(0,0)$ to $(2,0)$, add it to the curve, then subtract off $\int_L \mathbf{F}\cdot d\mathbf{r}$]

With L:

$$\oint_{C\cup L} F\cdot dr \underset{\text{Green's}}{=} \iint \left[\frac{\partial}{\partial x}(3x+2xy) - \frac{\partial}{\partial y}(x+y^2)\right] dA$$

$$= \iint (3+2y-2y)\,dA$$

$$= 3\iint dA = 3\cdot 13 = 39$$

Thus,

$$\int_C \vec{F}\cdot d\vec{r} = 39 - \int_L \vec{F}\cdot d\vec{r} = 39 - \int_0^2 \langle t, 3t\rangle \cdot \frac{d}{dt}\langle t, 0\rangle\,dt$$

$$= 39 - \int_0^2 t\,dt = 39 - \left[\tfrac{1}{2}t^2\right]_0^2 = 39 - 2 = 37$$

$\int_C \mathbf{F}\cdot d\mathbf{r} =$ 37

19. [10 points] For the vector field $\mathbf{F}(x,y) = (1 - ye^{-x})\mathbf{i} + e^{-x}\mathbf{j}$, find $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is any path starting at $(0,1)$ and ending at $(1,2)$.

This vector field is conservative

$$\varphi(x,y) = x + ye^{-x} + g(y)$$

$$\varphi(x,y) = ye^{-x} + f(x)$$

Thus,

$$\varphi(x,y) = x + ye^{-x}$$

is a potiential function

$$\int \vec{F} \cdot d\vec{r} = \varphi(1,2) - \varphi(0,1)$$

$$= 1 + 2e^{-1} - 1$$

$$= \frac{2}{e}$$

$\int_C \mathbf{F} \cdot d\mathbf{r} = \frac{2}{e}$

20. [10 points] Let $f(x, y)$ be a differentiable function such that

$$\left.\frac{\partial f}{\partial x}\right|_{(x,y)=(2,2)} = -3; \quad \left.\frac{\partial f}{\partial y}\right|_{(x,y)=(2,2)} = 5$$

If $x = r\cos\theta$ and $y = r\sin\theta$, find $\frac{\partial f}{\partial \theta}$ at the point $(r, \theta) = \left(2\sqrt{2},\ \frac{\pi}{4}\right)$.

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$\frac{\partial x}{\partial \theta} = -r\sin\theta$ $\qquad$ $\frac{\partial y}{\partial \theta} = r\cos\theta$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta}$$

$$= -r\sin\theta\,\frac{\partial f}{\partial x} + r\cos\theta\,\frac{\partial f}{\partial y}$$

Evaluate at $(r,\theta) = (2\sqrt{2}, \frac{\pi}{4})$, then $(x,y) = (2,2)$

$$\left.\frac{\partial f}{\partial \theta}\right|_{(2\sqrt{2},\frac{\pi}{4})} = -2\sqrt{2}\sin\frac{\pi}{4}\cdot\left.\frac{\partial f}{\partial x}\right|_{(2,2)} + 2\sqrt{2}\cos\frac{\pi}{4}\left.\frac{\partial f}{\partial y}\right|_{(2,2)}$$

$$= (-2\sqrt{2})\left(\frac{\sqrt{2}}{2}\right)(-3) + (2\sqrt{2})\left(\frac{\sqrt{2}}{2}\right)(5)$$

$$= 6 + 10$$

$$= 16$$

$$\left.\frac{\partial f}{\partial \theta}\right|_{(r,\theta)=\left(2\sqrt{2},\ \frac{\pi}{4}\right)} = 16$$