MA 261 ONLINE, SUMMER 2026
Quiz 10: §17.6 (Part III), 17.7 (Part I)

Name: Key
PUID:

Score: /20
Length: 15 minutes

Directions: Follow these instructions carefully. Failure to do so may result in your quiz being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.

- Complete each question below. There are 3 questions; the maximum score is 20 points.
- You have 25 minutes: 15 minutes to complete the quiz and 10 minutes to upload it to Gradescope.
- This quiz is closed book and closed notes. Calculators are not allowed.
- You are to work independently. External help (including, but not limited to, using the internet and generative AI) is prohibited.
- You may use your own paper to complete this quiz; you do not need to print this document.
- Adequate work must be shown for each problem. A question with little to no work, even with a correct answer, will result in a low score.
- Make sure your work is neat and legible, especially after it is scanned. Use proper notation, and clearly mark the question numbers and your final answer. Points may be taken off if part of your work is unreadable.

(The questions start on the next page, one problem per page)

1. [10 points] Set up an integral the computes the flux of the vector field $\vec{F}(x,y,z) = xze^y\mathbf{i} - xze^y\mathbf{j} + z\mathbf{k}$ across the surface S, where S is the part of the plane $x+y+z=1$ in the first octant and has upward orientation.

   Simplify your answer, but **do not evaluate the integral.**

Plane $\vec{P} = \langle x, y, 1-x-y \rangle$

$P_x = \langle 1, 0, -1 \rangle$

$P_y = \langle 0, 1, -1 \rangle$

$$P_x \times P_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = \vec{i} + \vec{j} + \vec{k} = \langle 1, 1, 1 \rangle$$

$$\text{Flux} = \int \vec{F} \cdot (P_x \times P_y)\, dA = \int \langle xze^y, -xze^y, z \rangle \cdot \langle 1, 1, 1 \rangle\, dA$$

$$= \int z\, dA$$

$$= \int_{x=0}^{1} \int_{y=0}^{1-x} (1-x-y)\, dy\, dx$$

2. [9 points] Use Stokes' Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F}(x, y, z) = \langle yz,\ 2xz,\ e^{xy} \rangle$ and C is the circle $x^2 + y^2 = 16$ in the plane $z = 5$ oriented counterclockwise as viewed from above.

$$\text{Curl}(\vec{F}) = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ yz & 2xz & e^{xy} \end{vmatrix} = (xe^{xy} - 2x)\vec{i} - (ye^{xy} - y)\vec{j} + (2z - z)\vec{k}$$

$$\oint_C F \cdot d\vec{r} = \int_S \text{Curl}\,\vec{F} \cdot d\vec{S}$$

$$= \int \text{curl}\,\vec{F} \cdot \langle 0, 0, 1 \rangle \, dA$$

$$= \int z \, dA$$

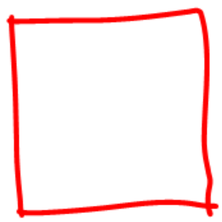
$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{4} 5r \, dr \, d\theta = 10\pi \cdot \frac{1}{2} r^2 \Big]_0^4$$

$$= 80\pi$$

3. [1 point] Final quiz question of the semester: What's 2+2?

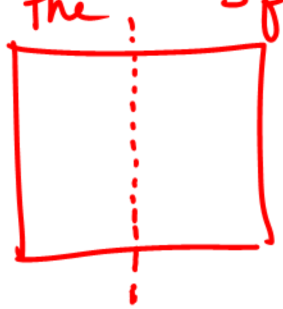
(Answers that attempt to make the grader laugh are highly encouraged.)

4, but where is the fun without a silly and needlessly convoluted explanation. Given a square



We count in two ways the number of ways of selecting a corner. The total number of corners is 4, hence there are 4 ways we can choose a corner.

However, split the square in half



There are 2 ways of selecting a corner left of the divide, and 2 ways of selecting a corner to the right of the divide.

Thus, there are 2+2 ways of selecting a corner of the square. Therefore,

$$4 = 2+2$$