MA 261 ONLINE, SUMMER 2026

Quiz 3: §14.3-14.5, 15.1-15.2

Name: Key

PUID:

Score: /~~15~~ 20

Length: 15 minutes

Directions: Follow these instructions carefully. Failure to do so may result in your quiz being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.

- Complete each question below. There are 3 questions; the maximum score is ~~15~~ 20 points.
- You have 25 minutes: 15 minutes to complete the quiz and 10 minutes to upload it to Gradescope.
- This quiz is closed book and closed notes. Calculators are not allowed.
- You are to work independently. External help (including, but not limited to, using the internet and generative AI) is prohibited.
- You may use your own paper to complete this quiz; you do not need to print this document.
- Adequate work must be shown for each problem. A question with little to no work, even with a correct answer, will result in a low score.
- Make sure your work is neat and legible, especially after it is scanned. Use proper notation, and clearly mark the question numbers and your final answer. Points may be taken off if part of your work is unreadable.

(The questions start on the next page, one problem per page)

1. [~~4~~ 8 points] Find the length of the curve described by $\vec{r}(t) = \langle 3\cos 2t,\ 5\sin 2t,\ 4\cos 2t\rangle$ where $0 \le t \le \frac{\pi}{5}$.

$$\vec{r}\,'(t) = \langle 2\cdot 3\cdot(-\sin 2t),\ 2\cdot 5\cdot \cos 2t,\ 2\cdot 4\cdot(-\sin 2t)\rangle$$

$$= \langle -6\sin 2t,\ 10\cos 2t,\ -8\sin 2t\rangle$$

$$|\vec{r}\,'(t)| = \sqrt{36\sin^2 2t + 100\cos^2 2t, 64\sin^2 2t} = \sqrt{100} = 10$$

$$L = \int_0^{\pi/5} |\vec{r}\,'(t)|\, dt = \int_0^{\pi/5} 10\, dt = 10\left(\frac{\pi}{5}\right) = \boxed{2\pi}$$

2. [~~4~~ 6 points] Find all values of t (if any) where the curve $\vec{r}(t) = \langle 2-t, 3, t+7 \rangle$ has 0 curvature.

Based on the formula used

$\vec{r}'(t) = \langle -1, 0, 1 \rangle$

$T(t) = \frac{1}{|\vec{r}'(t)|} \langle -1, 0, 1 \rangle$

$= \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$

$\Rightarrow T'(t) = \langle 0, 0, 0 \rangle$

$\Rightarrow K = \frac{|T'(t)|}{|\vec{r}'(t)|} = 0$

$\Rightarrow K = 0$ for any value of t

$\vec{v}(t) = \langle -1, 0, 1 \rangle$

$\vec{a}(t) = \langle 0, 0, 0 \rangle$

$\vec{v} \times \vec{a} = \vec{0}$

$\Rightarrow K = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3} = \frac{0}{(\sqrt{2})^3} = 0$

$\Rightarrow K = 0$ for any value of t

3. [6 points] Compute the limit below, or show it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{2x^2+y^2}$$

We claim the limit DNE, suppose it did and call it L.

Computing the limit along the path $y=x$

$$L = \lim_{x\to 0} \frac{x(x)}{2x^2+(x)^2} = \lim_{x\to 0} \frac{x^2}{3x^2} = \lim_{x\to 0} \frac{1}{3} = \frac{1}{3}$$

But computing it along the path $y=-x$

$$L = \lim_{x\to 0} \frac{x(-x)}{2x^2+(-x)^2} = \lim_{x\to 0} -\frac{x^2}{3x^2} = \lim_{x\to 0} -\frac{1}{3} = -\frac{1}{3} \neq \frac{1}{3}$$

which is nonsense

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{xy}{2x^2+y^2}$$ DNE by the Two Paths Test