MA 261 ONLINE, SUMMER 2026
Quiz 7: §16.5, 16.6, 17.1

Name: Key
PUID:

Score: /20
Length: 15 minutes

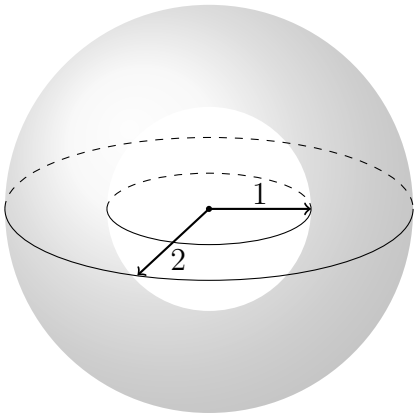
Directions: Follow these instructions carefully. Failure to do so may result in your quiz being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.

- Complete each question below. There are 2 questions; the maximum score is 20 points.
- You have 25 minutes: 15 minutes to complete the quiz and 10 minutes to upload it to Gradescope.
- This quiz is closed book and closed notes. Calculators are not allowed.
- You are to work independently. External help (including, but not limited to, using the internet and generative AI) is prohibited.
- You may use your own paper to complete this quiz; you do not need to print this document.
- Adequate work must be shown for each problem. A question with little to no work, even with a correct answer, will result in a low score.
- Make sure your work is neat and legible, especially after it is scanned. Use proper notation, and clearly mark the question numbers and your final answer. Points may be taken off if part of your work is unreadable.

(The questions start on the next page, one problem per page)

1. [10 points] A spherical shell with an inner radius of 1 and an outer radius of 2 is placed at the origin (see diagram below). If the density at the point (x, y, z) is $\sqrt{x^2 + y^2 + z^2}$, set up an integral to calculate the mass of the shell.

   Do NOT evaluate the integral.



$$M = \int_{\text{Shell}} \sqrt{x^2+y^2+z^2}\, dV = \int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\pi} \int_{\rho=1}^{2} \rho \cdot \rho^2 \sin\varphi \; d\rho \, d\varphi \, d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\pi} \int_{\rho=1}^{2} \rho^3 \sin\varphi \; d\rho \, d\varphi \, d\theta$$

2. Let $\vec{F}(x,y) = \langle 2x, -1\rangle$ and $C = \{(x,y) |\ y - x^2 = 1\}$.

(a) [4 points] Determine the points (if any) along the curve C at which the vector field $\vec{F}$ is tangent to C.

Suppose C is the level curve $g \equiv 1$ where $g = y - x^2$

Then $\vec{F}$ is tangent to C if $\vec{F} \perp \nabla g$

$\nabla g = \langle -2x, 1\rangle$. Solve

$$\langle 2x, -1\rangle \cdot \langle -2x, 1\rangle = 0$$

$$-4x^2 - 1 = 0$$

$$x^2 = -\tfrac{1}{4}. \text{ No Solution!}$$

Therefore $\vec{F}$ is never tangent

(b) [4 points] Show that $\vec{F}$ is normal to C at **every** point along C.

Similarly, $\vec{F}$ is normal if it is parallel to ∇g. Le there is λ such that

$$\vec{F} = \lambda \nabla g$$

$$\langle 2x, -1\rangle = \lambda \langle -2x, 1\rangle$$

$$\Rightarrow \begin{cases} 2x = -2\lambda x \\ -1 = \lambda \end{cases} \Rightarrow 2x = 2x \leftarrow \text{True for any } x$$

Therefore it is normal at every part

(c) [2 points] Sketch C and a few representatives of $\vec{F}$ on C.