MA 261 ONLINE, SUMMER 2026
Quiz 9: §17.6 (Parts I and II)

Name: Key
PUID:

Score: /20
Length: 15 minutes

Directions: Follow these instructions carefully. Failure to do so may result in your quiz being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.

- Complete each question below. There are 2 questions; the maximum score is 20 points.
- You have 25 minutes: 15 minutes to complete the quiz and 10 minutes to upload it to Gradescope.
- This quiz is closed book and closed notes. Calculators are not allowed.
- You are to work independently. External help (including, but not limited to, using the internet and generative AI) is prohibited.
- You may use your own paper to complete this quiz; you do not need to print this document.
- Adequate work must be shown for each problem. A question with little to no work, even with a correct answer, will result in a low score.
- Make sure your work is neat and legible, especially after it is scanned. Use proper notation, and clearly mark the question numbers and your final answer. Points may be taken off if part of your work is unreadable.

(The questions start on the next page, one problem per page)

1. [10 points] Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

Parametrize the surface $f(x,y) = \langle x, y, xy \rangle$

We'll do it the long way here

$$f_x \times f_y = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & y \\ 0 & 1 & x \end{vmatrix} = -y\vec{i} - x\vec{j} + \vec{k}$$

$$|f_x \times f_y| = \sqrt{1 + x^2 + y^2}$$

$$\text{Area} = \int_{\text{Surface}} 1 \, dS = \int |f_x \times f_y| \, dA = \int \sqrt{1+x^2+y^2} \, dA$$

To Polar

$$\int_0^{2\pi} \int_0^1 \sqrt{1+r^2} \, r \, dr \, d\theta = 2\pi \int_0^1 \sqrt{1+r^2} \, r \, dr$$

$$= \pi \int_0^1 \sqrt{1+r^2} \, (2r) \, dr = \pi \cdot \frac{2}{3} (1+r^2)^{\frac{3}{2}} \Big]_0^1$$

$$= \frac{2\pi}{3} \left[2^{\frac{3}{2}} - 1^{\frac{3}{2}} \right] = \frac{2\pi}{3} \left[2\sqrt{2} - 1 \right]$$

2. [10 points] Find the mass of the part of the plane $z = 10 - 4x - y$ lying over the square $Q = \{(x, y)|\ 0 \le x \le 1,\ 0 \le y \le 1\}$ if the density at the point (x, y, z) is $\rho(x, y, z) = 12 - z$.

$$\rho(x,y) = 12 - (10 - 4x - y) = 2 + 4x + y$$

Using the shortcut on Chenflix

$$\text{Mass} = \int_{\text{Plate}} \rho\, dS = \int_{y=0}^{1}\int_{x=0}^{1} (2 + 4x + y)\sqrt{1 + \rho_x^2 + \rho_y^2}\, dx\, dy$$

$$= \int_0^1\int_0^1 (2 + 4x + y)\sqrt{1 + 4^2 + 1^2}\, dx\, dy = \sqrt{18}\int_0^1\int_0^1 (2 + 4x + y)\, dx\, dy$$

$$= \sqrt{18}\int_0^1 \left[2x + 2x^2 + xy\right]_{x=0}^{1} dy = \sqrt{18}\int_0^1 \left[2 + 2 + y\right] dy$$

$$= \sqrt{18}\left[4y + \frac{1}{2}y^2\right]_0^1 = \sqrt{18}\left[4 + \frac{1}{2}\right] = \frac{27}{2}\sqrt{2}$$